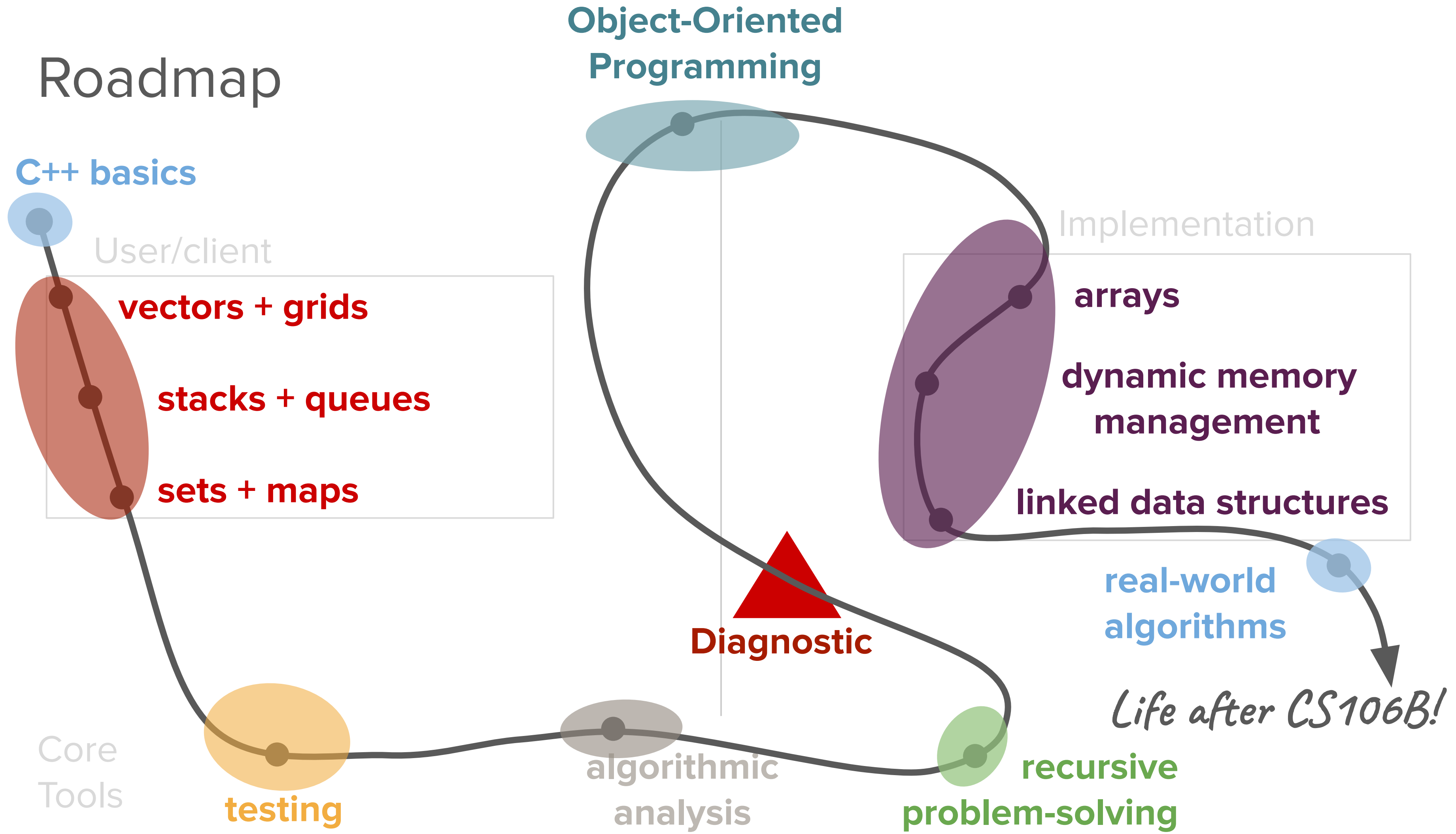
Trees

What are real-life examples of where you've seen sorting algorithms in action?

(put your answers the chat)

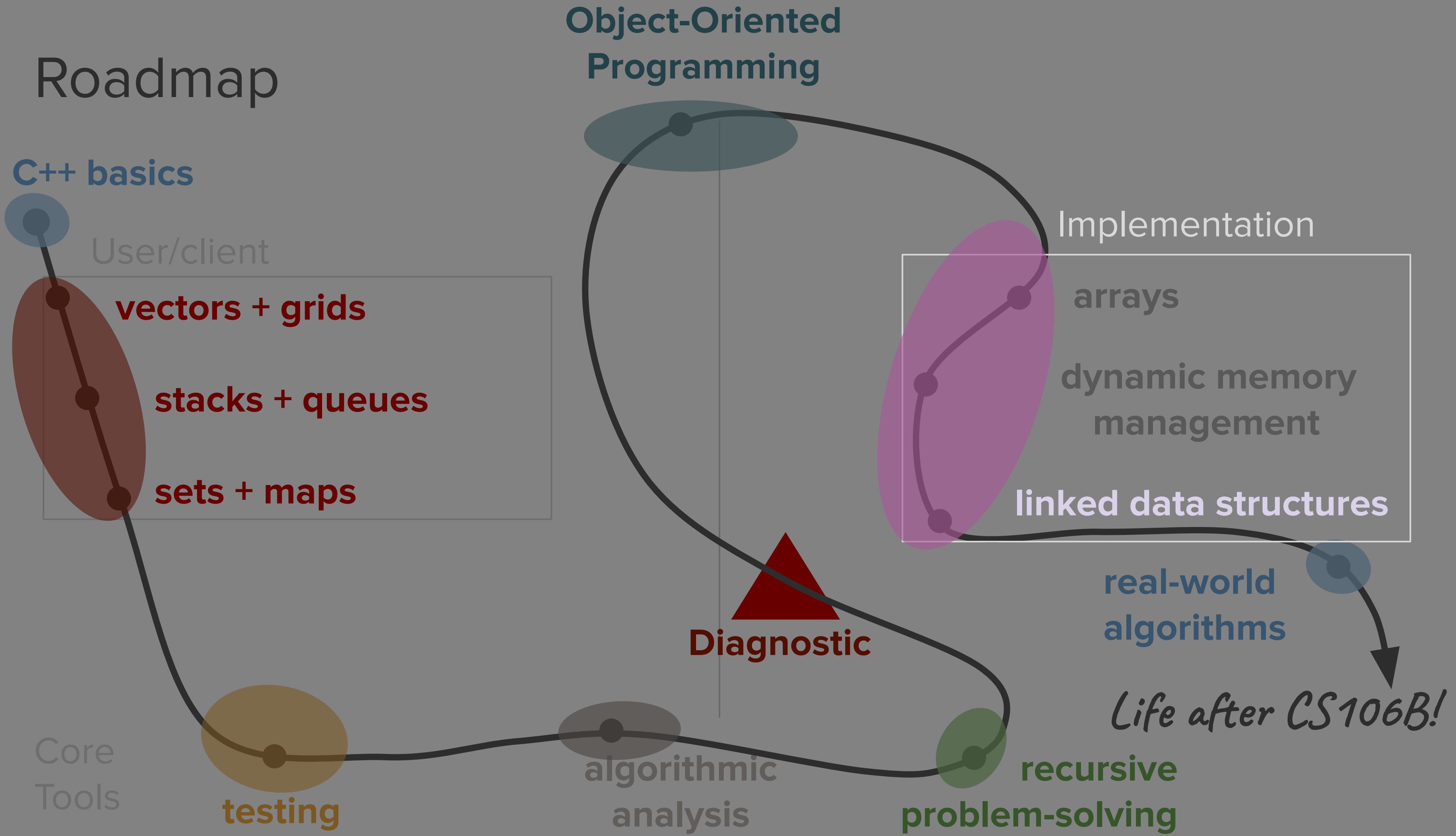
Roadmap


Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Today's topics

1. Sorting + Linked Data Structure Overview
2. Introduction to Trees
3. Trees in C++

Roadmap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Today's questions

How can we better organize data stored in a linked data structure?

Review

[sorting + linked data structures]

Sorting

- Sorting is a powerful tool for organizing data in a meaningful format!

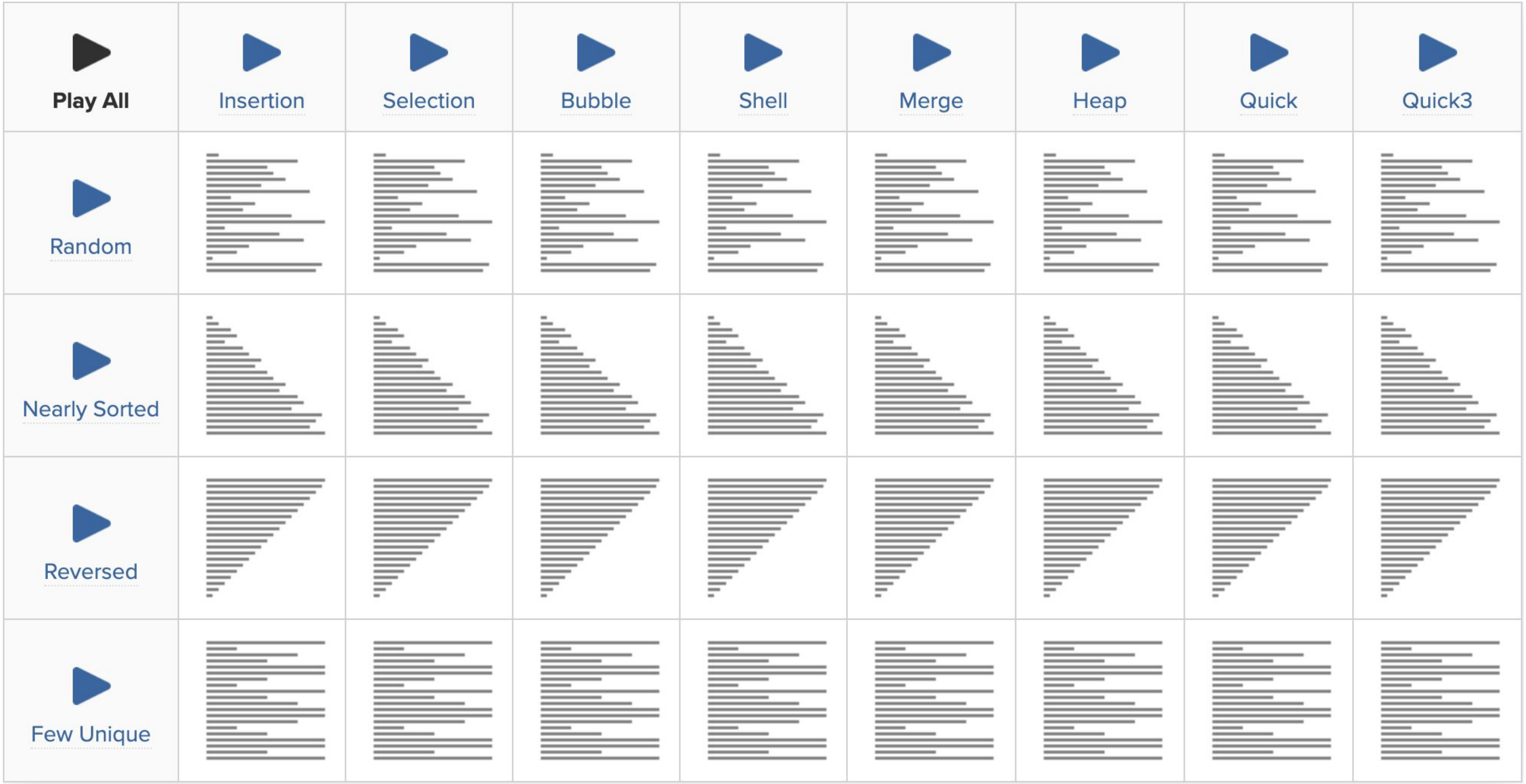
- There are many different methods for sorting data:
    - Selection Sort
    - Insertion Sort
    - Mergesort
    - Quicksort
    - And many more...

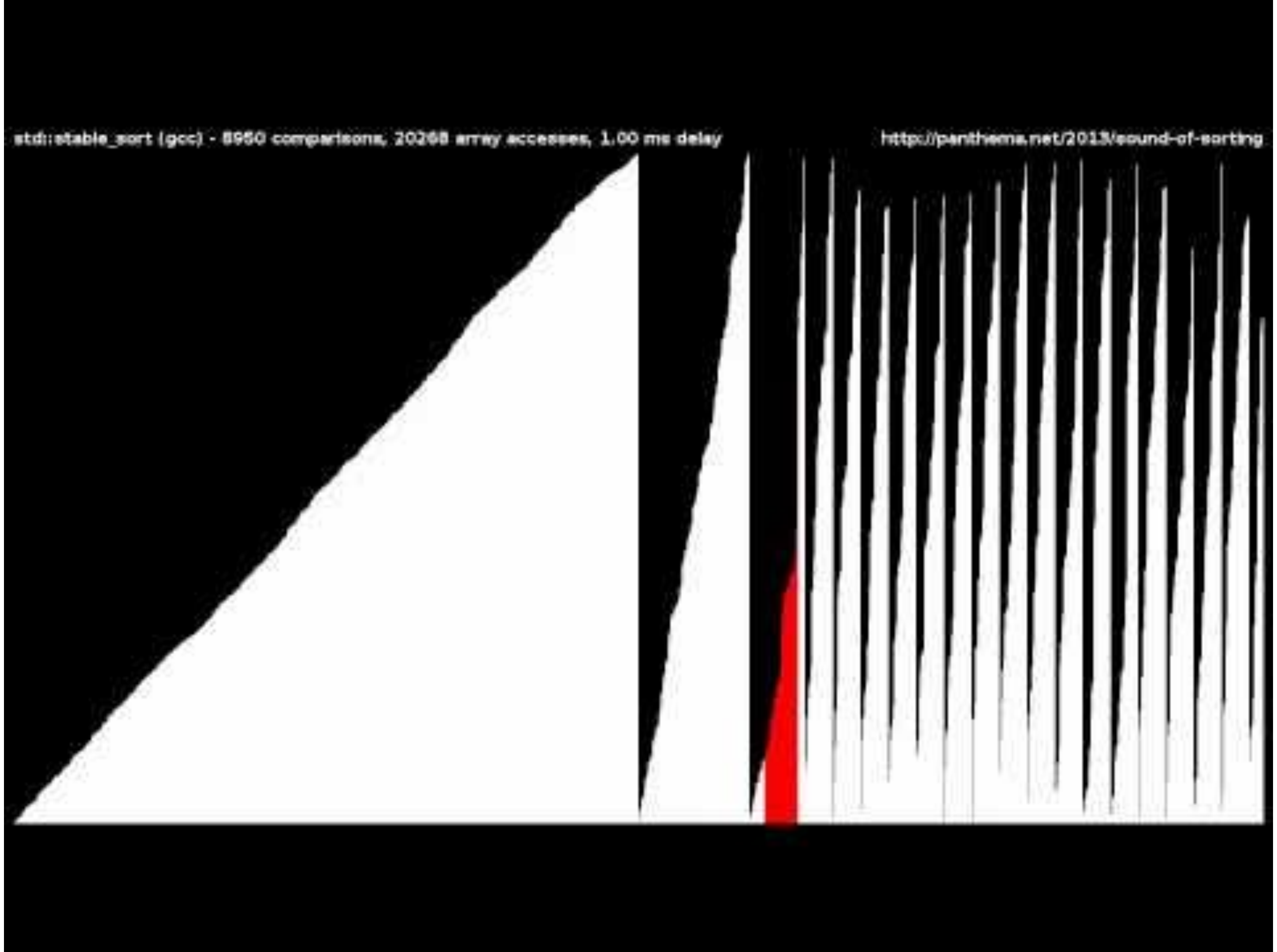
- Understanding the different runtimes and tradeoffs of the different algorithms is important when choosing the right tool for the job!

Sorting Big-O Cheat Sheet

Sort	Worst Case	Best Case	Average Case
Insertion	O(n^2)	O(n)	O(n^2)
Selection	O(n^2)	O(n^2)	O(n^2)
Merge	O(n log n)	O(n log n)	O(n log n)
Quicksort	O(n^2)	O(n log n)	O(n log n)

https://www.toptal.com/developers/sorting-algorithms




std::stable_sort (gcc) - 8950 comparisons, 20268 array accesses, 1.00 ms delay

http://panthema.net/2013/sound-of-sorting

Assignment 5:Linked List Tips

- When implementing the sorting algorithm on linked lists, it is strongly recommended to implement helper functions for the divide/join components of the algorithm.
  - For quicksort this means having helper functions for the partition and concatenate operations
- Everything you write should be implemented iteratively.
  - QuickSort is implemented recursively, but you’re only writing the individual components
  - For runSort, both the overall sort and the individual components should be done iteratively.
- Write tests for your helper functions first! Then, write end-to-end tests for your sorting function.

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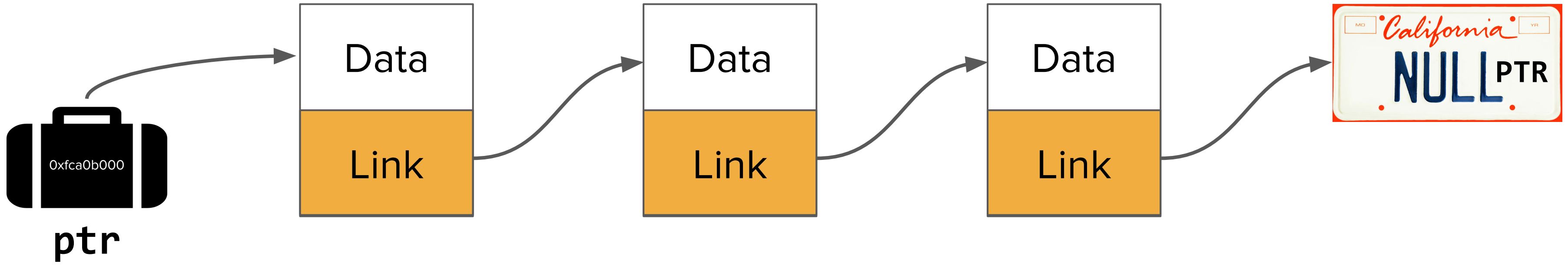

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- In order to organize this data, we had to **bundle data alongside pointers** in the concept of a "node."
- Using pointers allows us to **create links** to other nodes to impose structure.

Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
    - Insertion/removal of elements of a linked list was very quick because it only involved fast pointer rewiring operations. We never had to "shift" elements over to make room.
    - Because all the data was stored in dynamic memory, expanding the size of the linked list was very easy and never required an expensive "re-sizing" operation that had to copy all the data.

Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.

- However, we also ran into some limitations when it came to working with lists:
    - Data was organized in a linear structure, which meant the path to traverse between any two nodes (specifically between the front and a node later on in the list) could get very long.
    - Finding elements in a linked list is an **`O(n)`** operation, which can get slow when we want to store many elements.

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  - Finding elements in a linked list is an **`O(n)`** operation, which can get slow when we want to store many elements.
  - We couldn't feasibly write recursive algorithms that traversed linked lists, due to stack frame limits that came into play since traversal algorithms required one stack frame per node.

Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.

- However, we also ran into some limitations when it came to working with lists.

- **Question:** Can we organize data in a linked data structure in such a way that the path between the "front" and any element in the structure is short (better than `O(n)`) even if there are many elements?

How can we better organize data stored in a linked data structure?

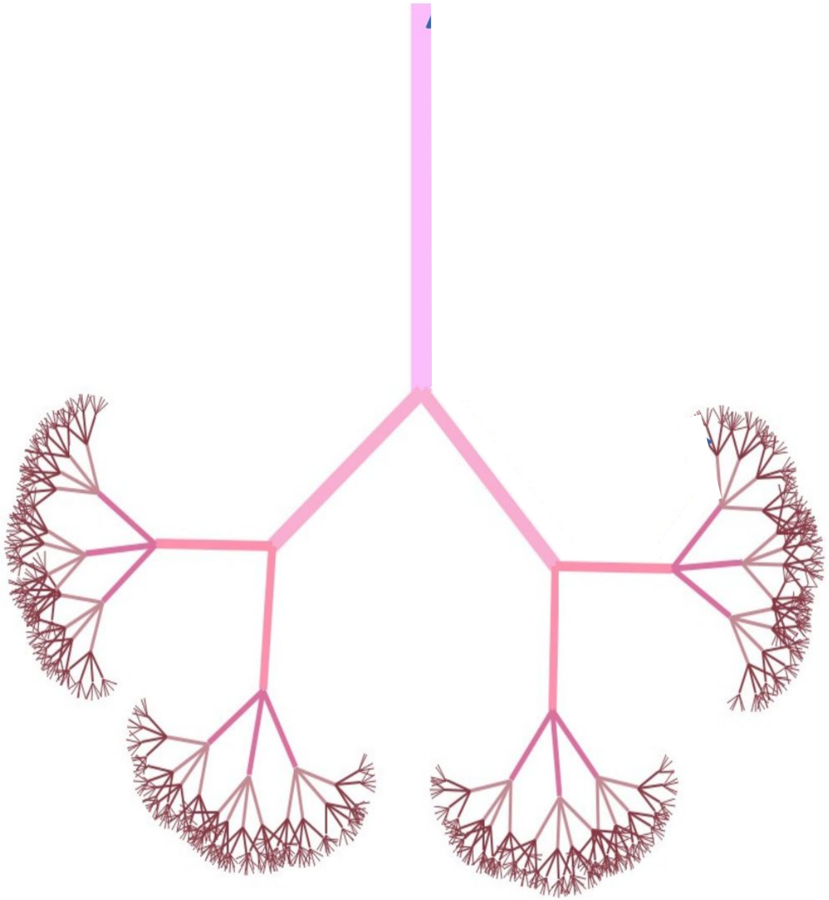
Interactive Exercise

[borrowed from Keith Schwarz]

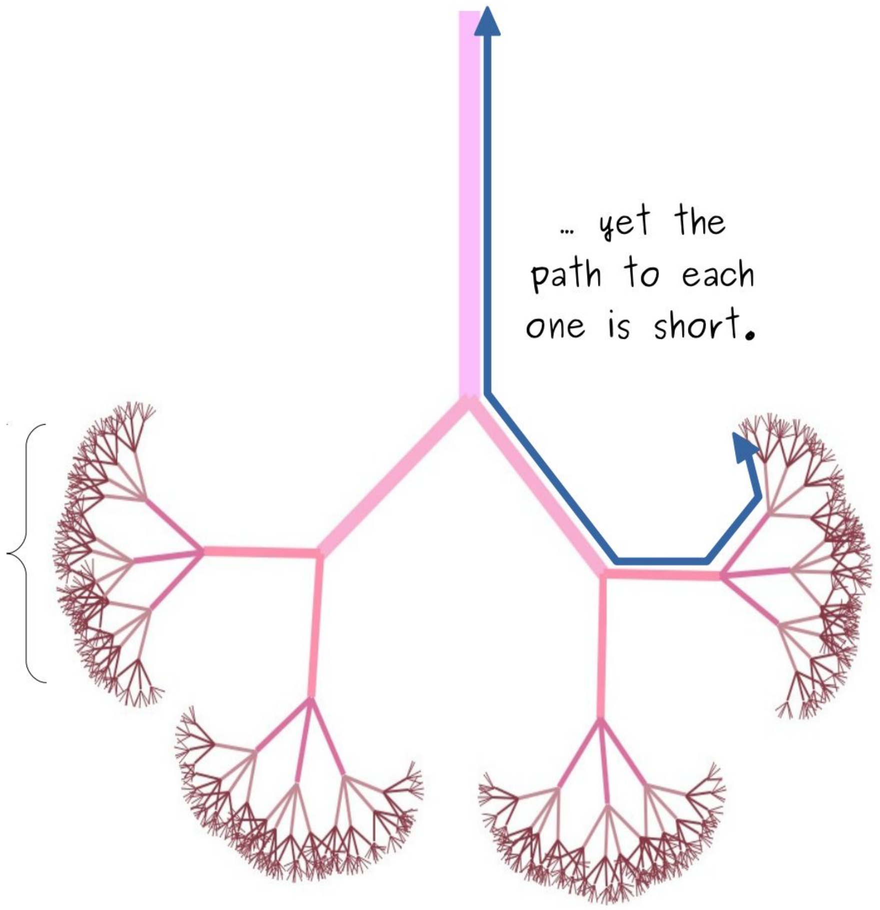
Take a deep breath.

And exhale...

Feel nicely oxygenated?



Your lungs have about 500 million alveoli...


Your lungs have about 500 million alveoli…
… yet the path to each one is short.

Key Idea: The distance from each element in this structure to the top of the structure is small, even if there are many elements.

Key Idea: branches

Trees

Throwback Thursday (on Monday)

- We've already seen trees before in this class... decision trees!

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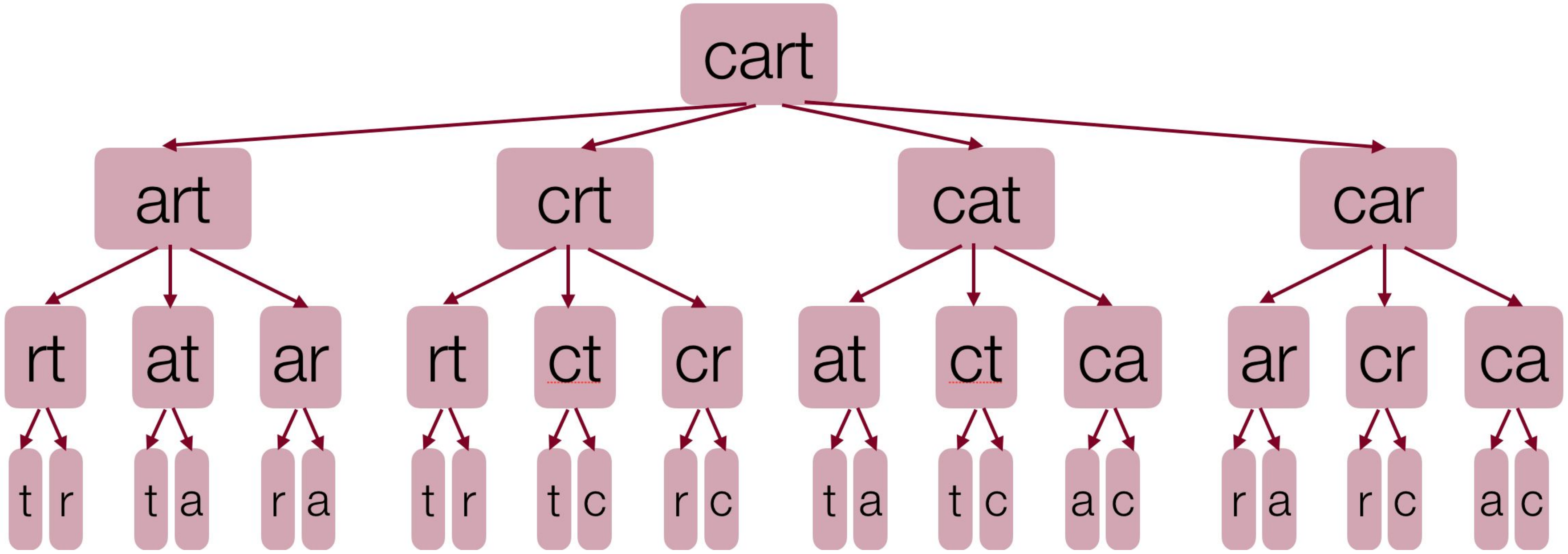

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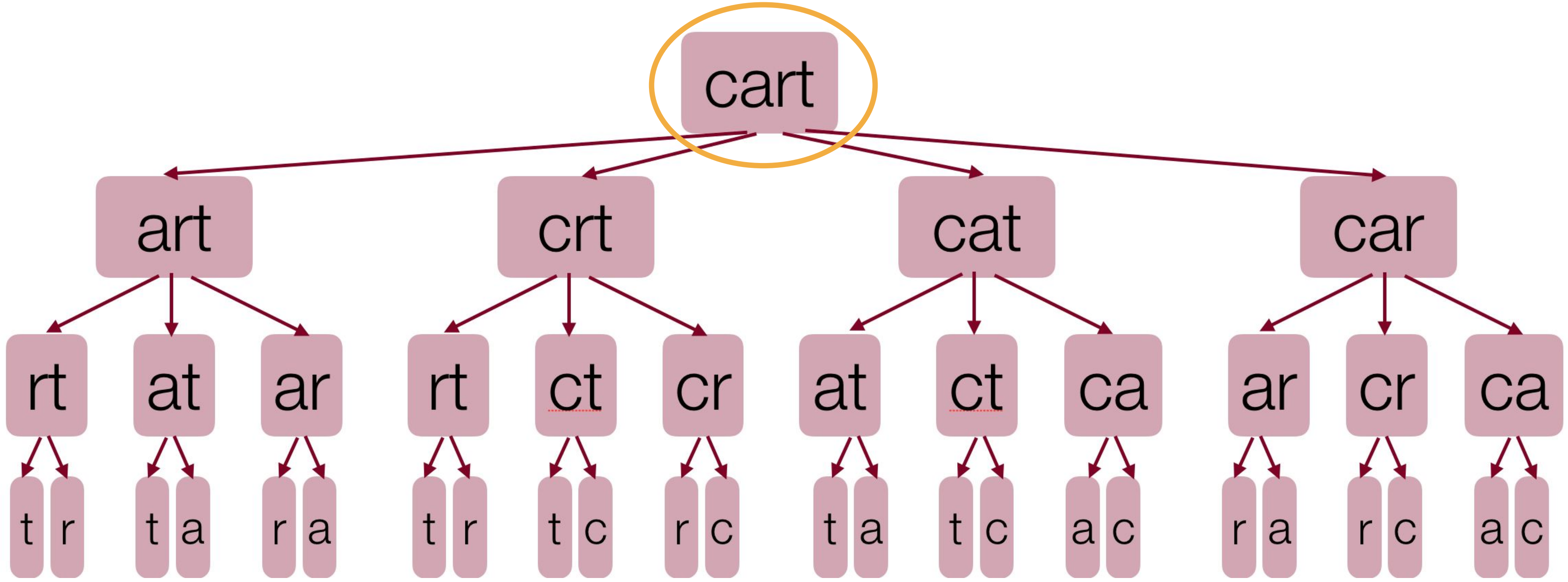

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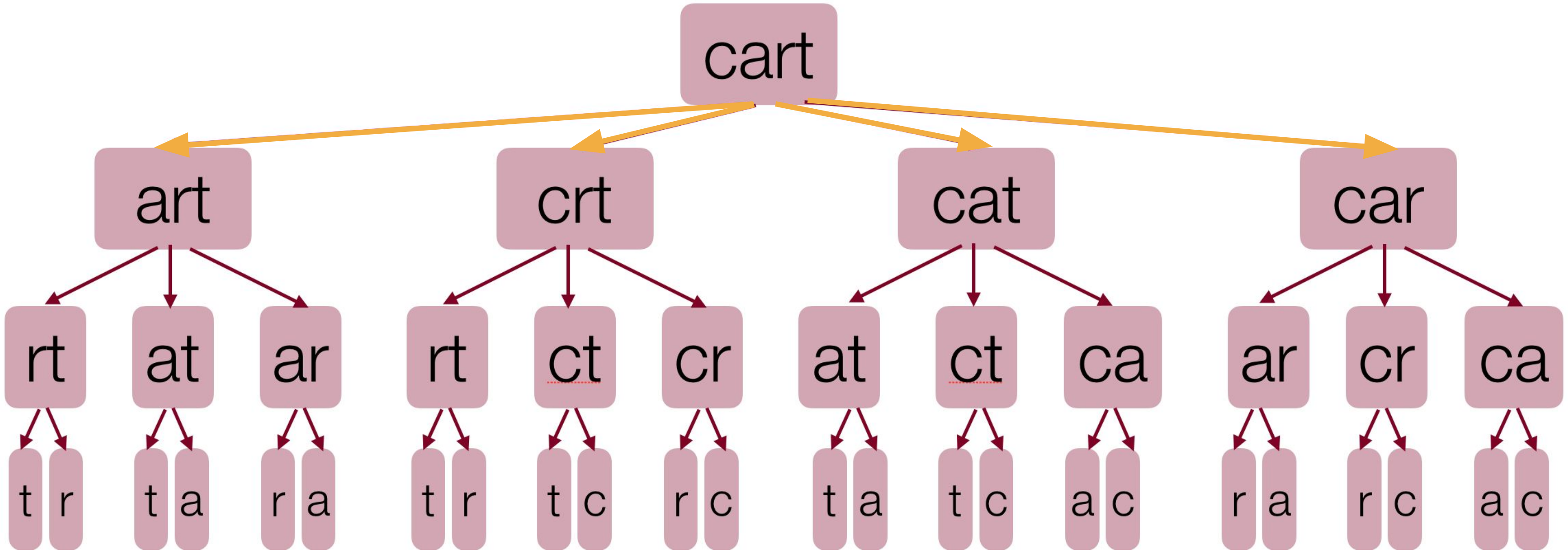

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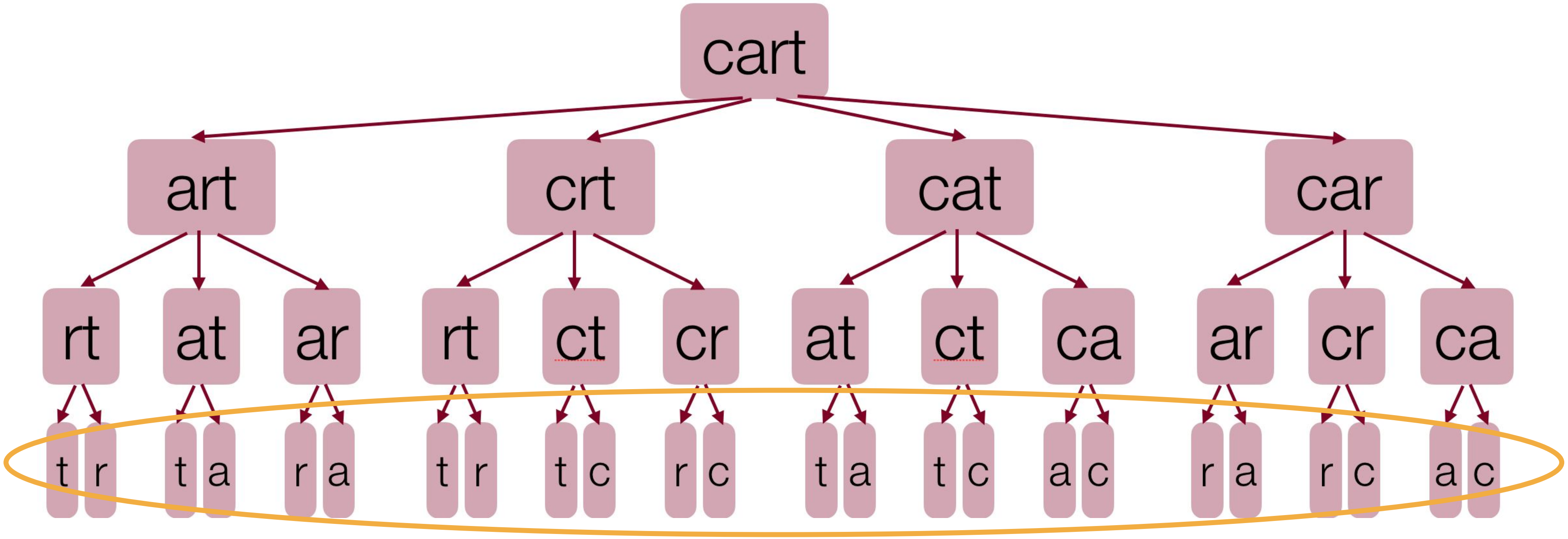

Trees in the Wild

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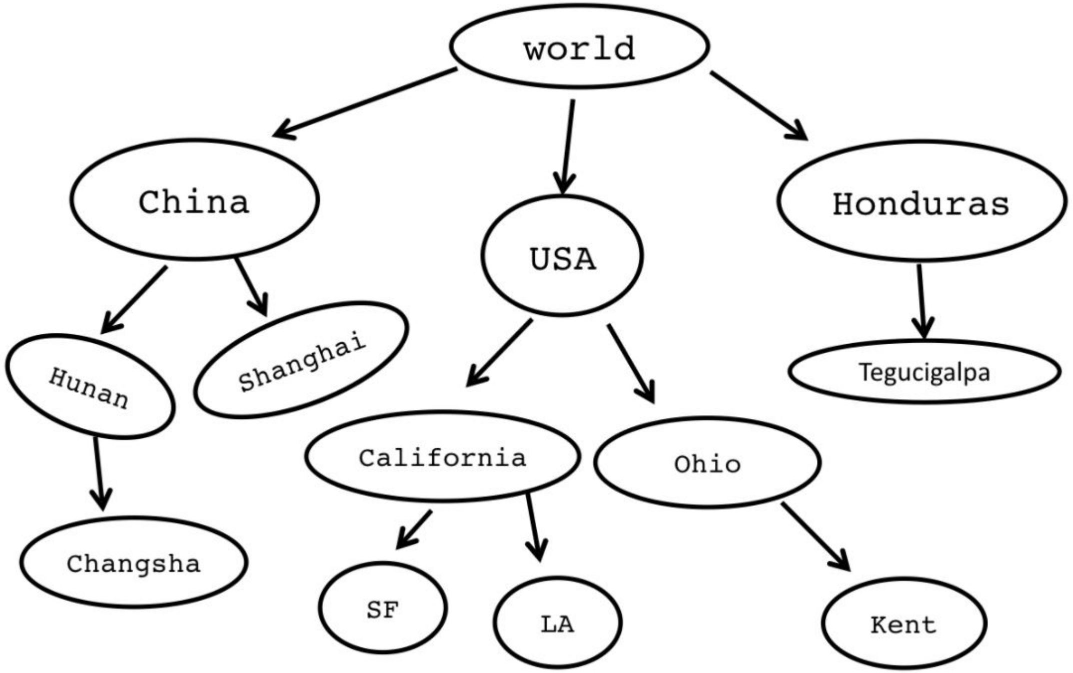

Trees can be used to describe *hierarchies*.

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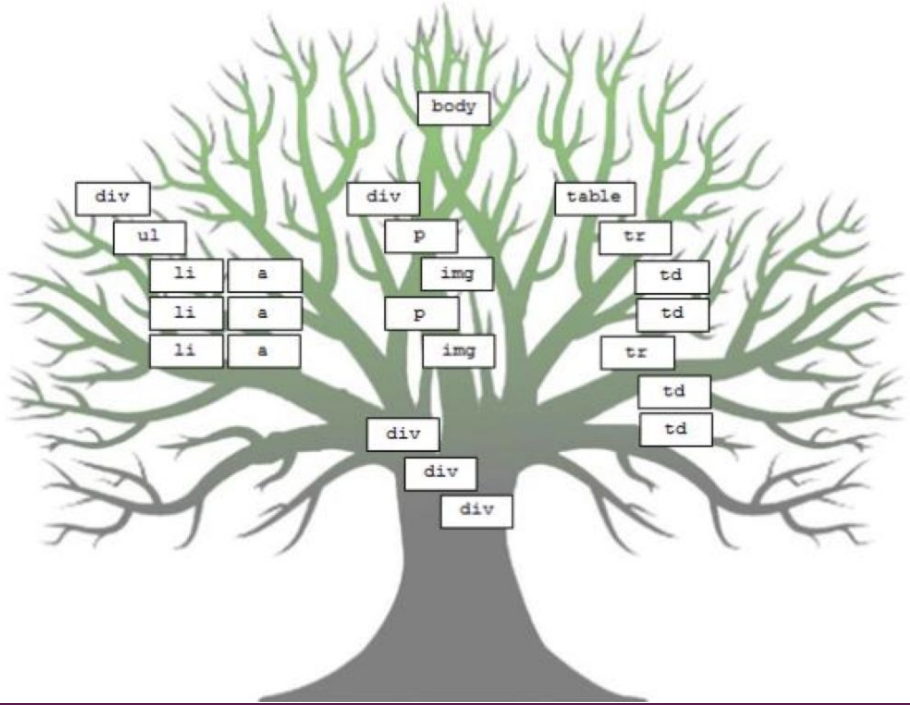
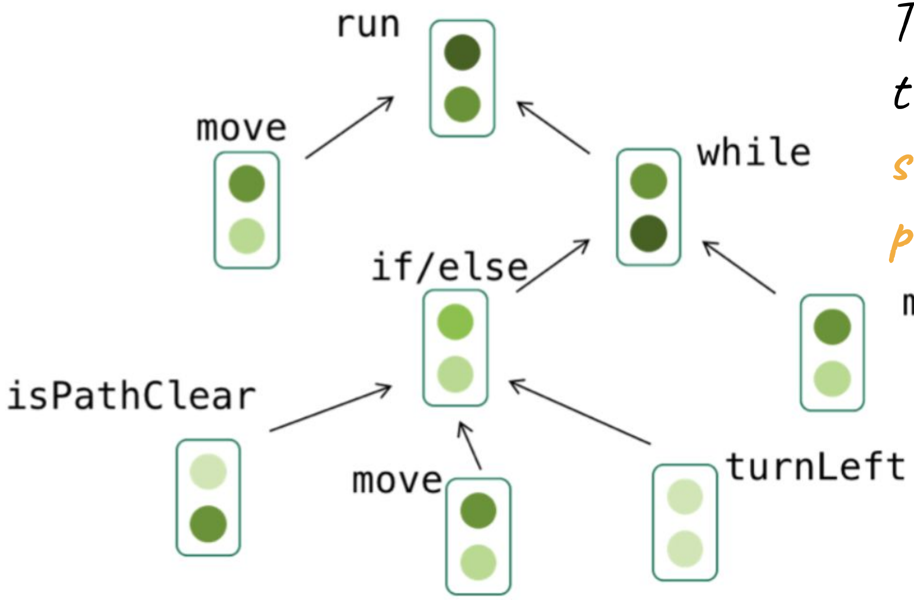


Trees are used to model the structure of websites.

Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

```
def run() {
  move();
  while (notFinished()) {
    if (isPathClear()) {
      move();
    } else {
      turnLeft();
    }
    move();
  }
}
```



Trees describe the syntax structure of programs.

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Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.
- But, it is not a coincidence that we first saw them appear in conjunction with recursion.
- Trees are inherently defined recursively!

What is a tree?

A tree is either...

What is a tree?

A tree is either...

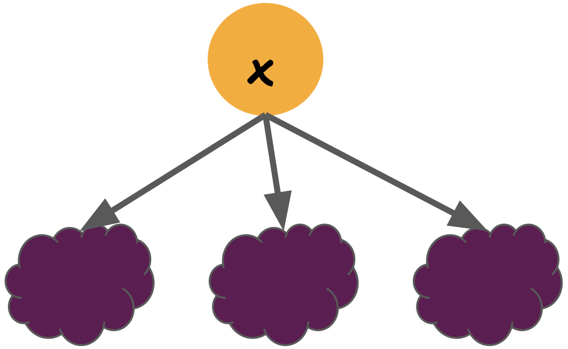
An empty data structure, or...

What is a tree?

A tree is either...

An empty data structure, or...

A single node (parent), with zero or more non-empty subtrees (children)



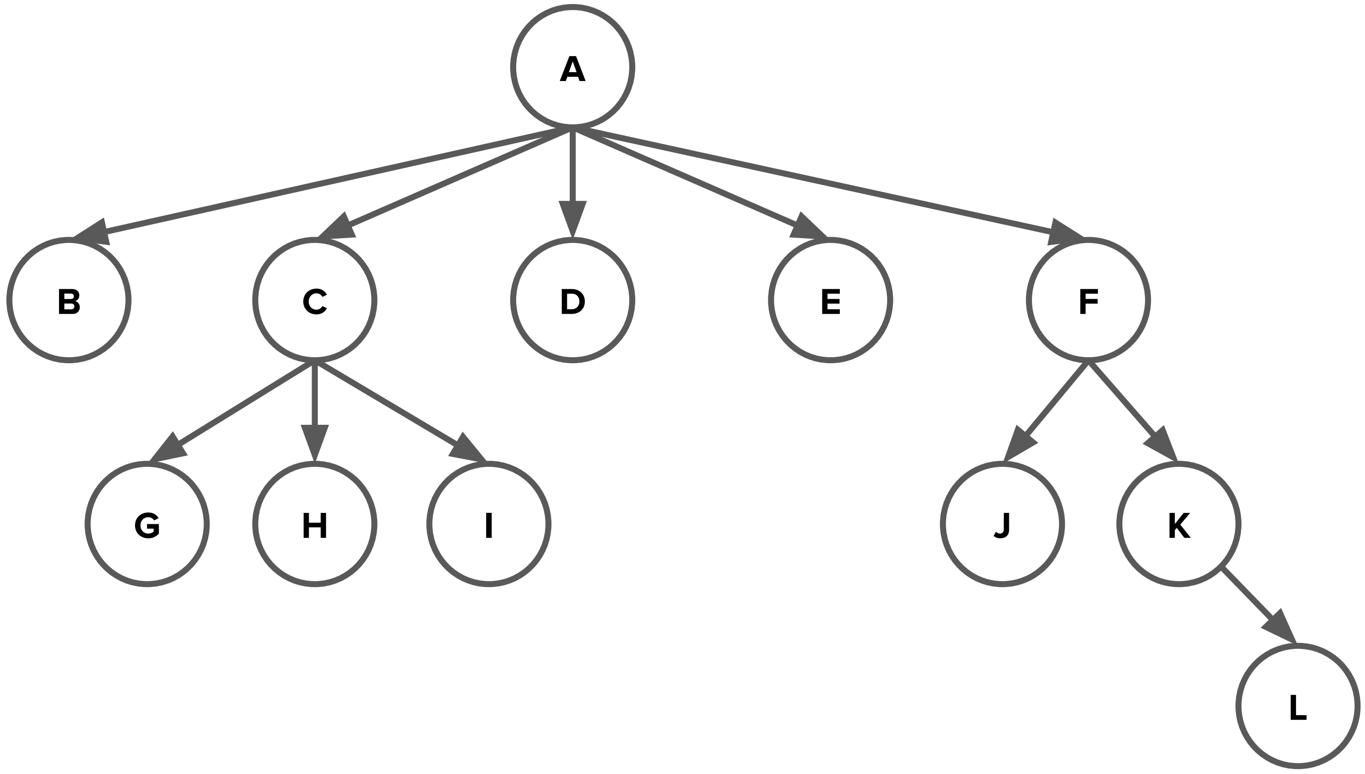
Definition

tree

A tree is hierarchical data organization structure composed of a root value linked to zero or more non-empty subtrees.

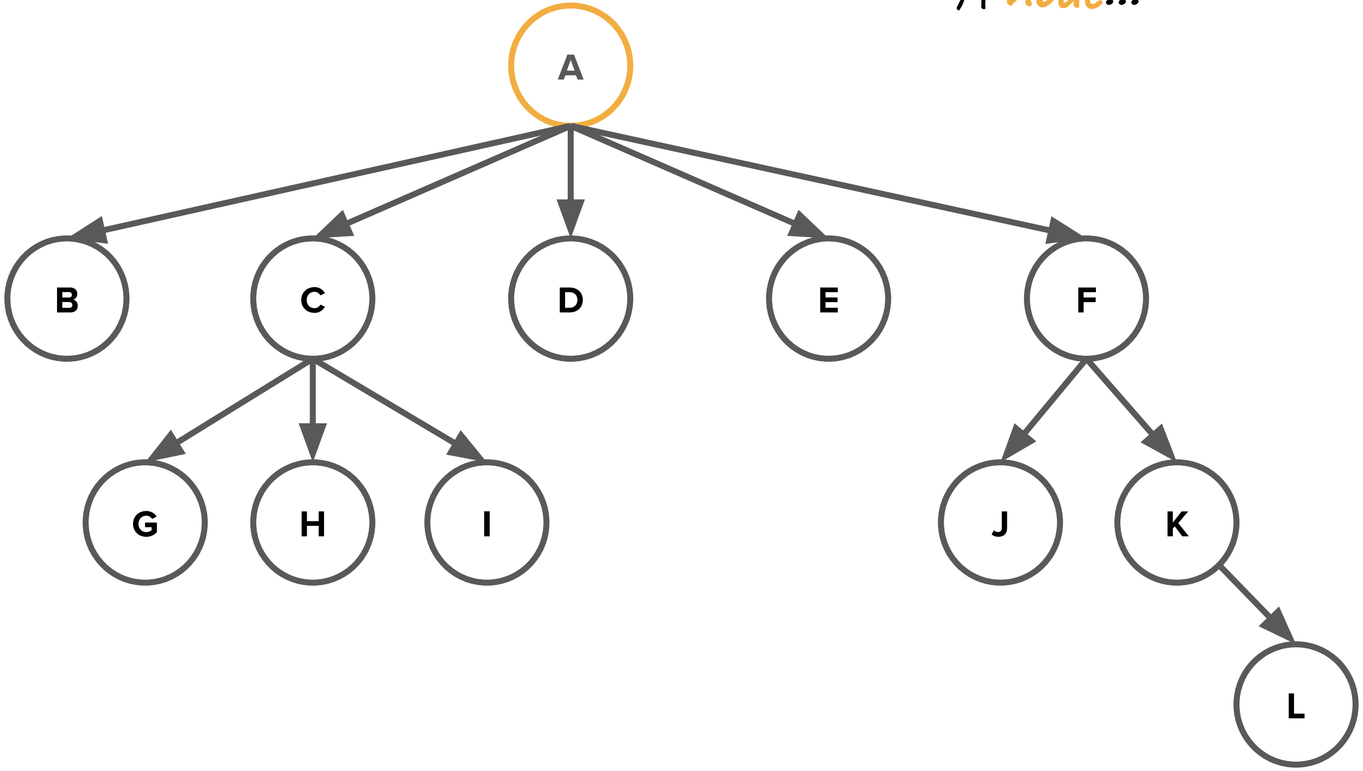
Tree Terminology

Tree Terminology



Tree Terminology

A node...



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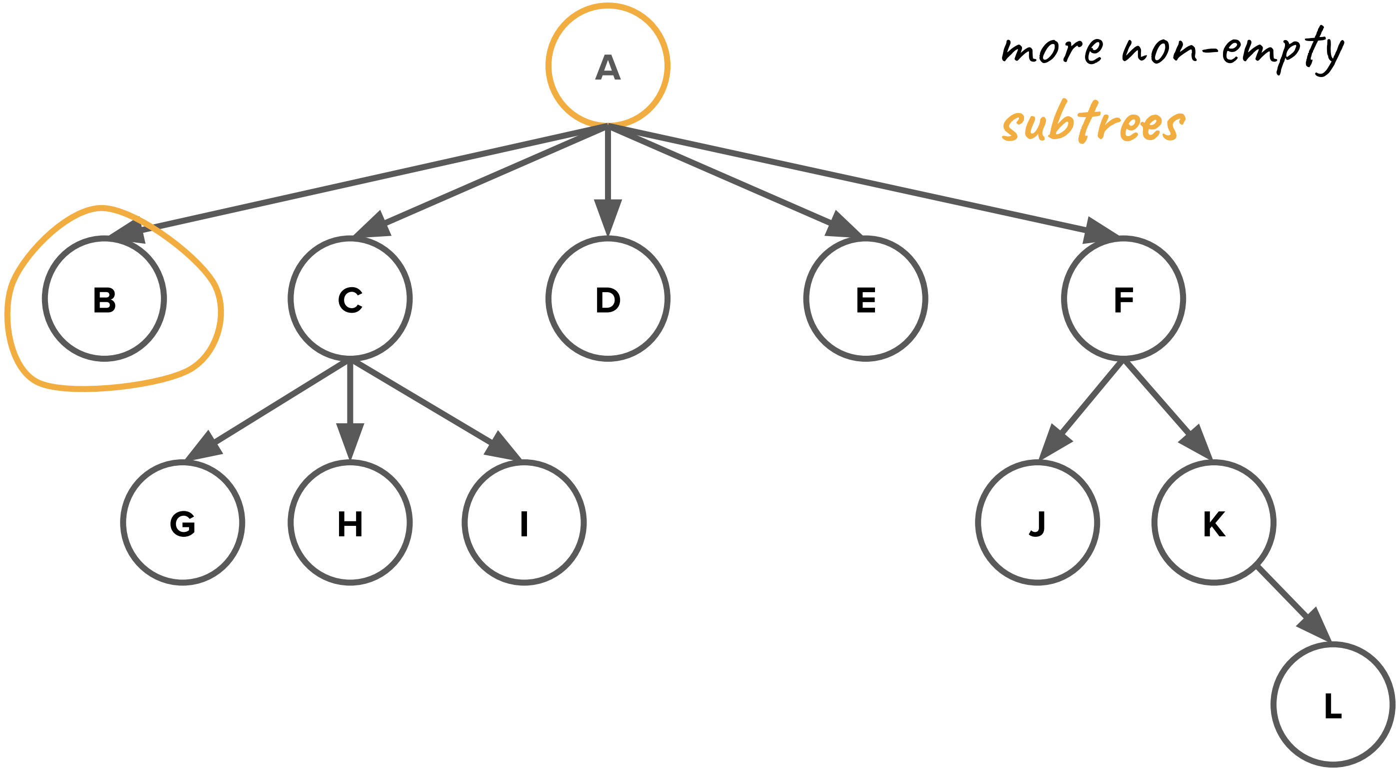
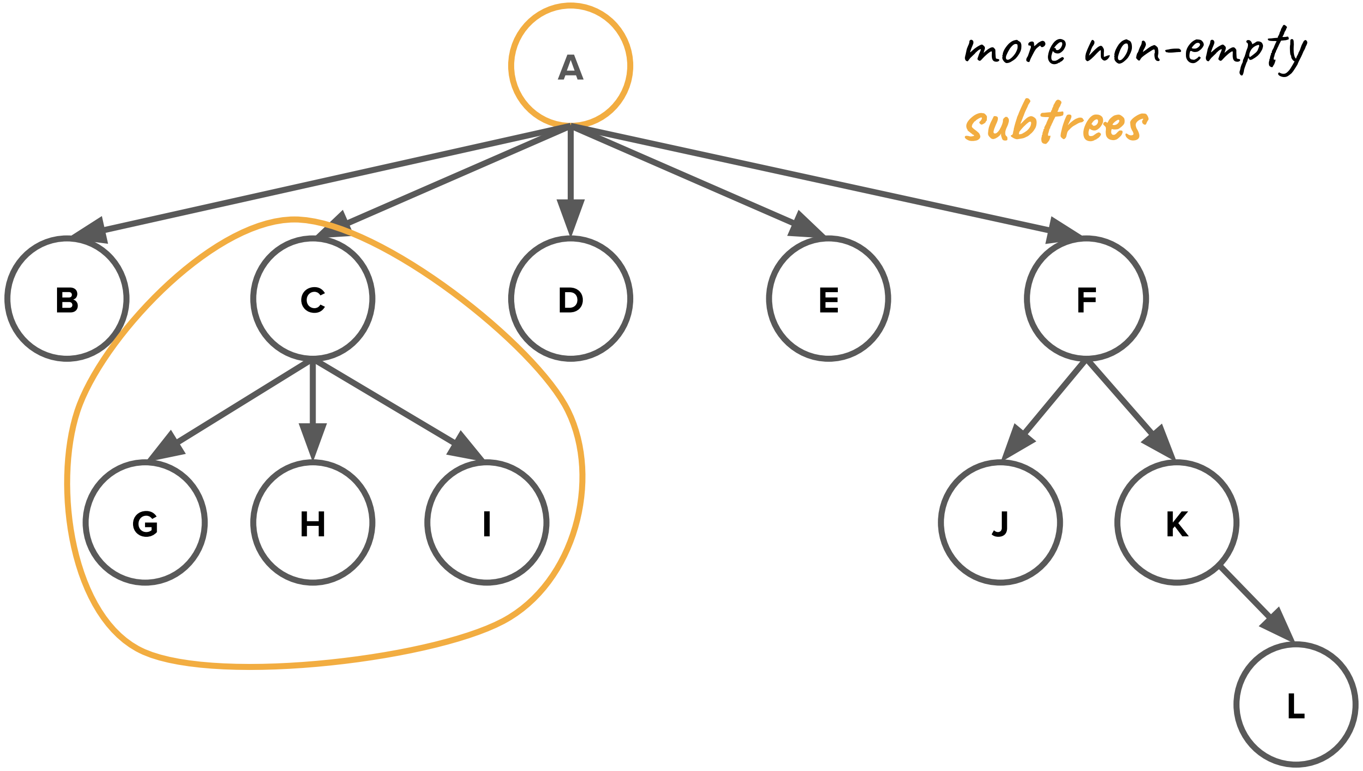


A node with 0 or more non-empty subtrees

Tree Terminology



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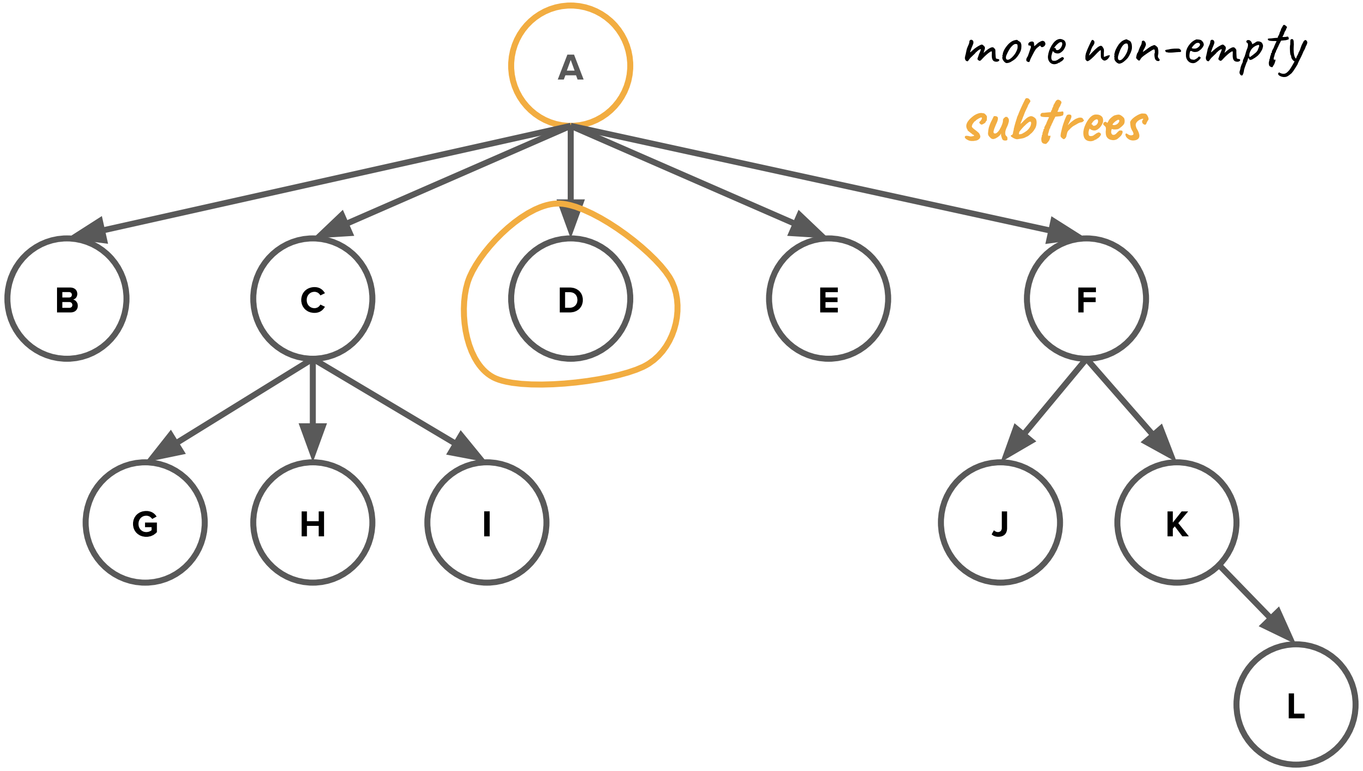

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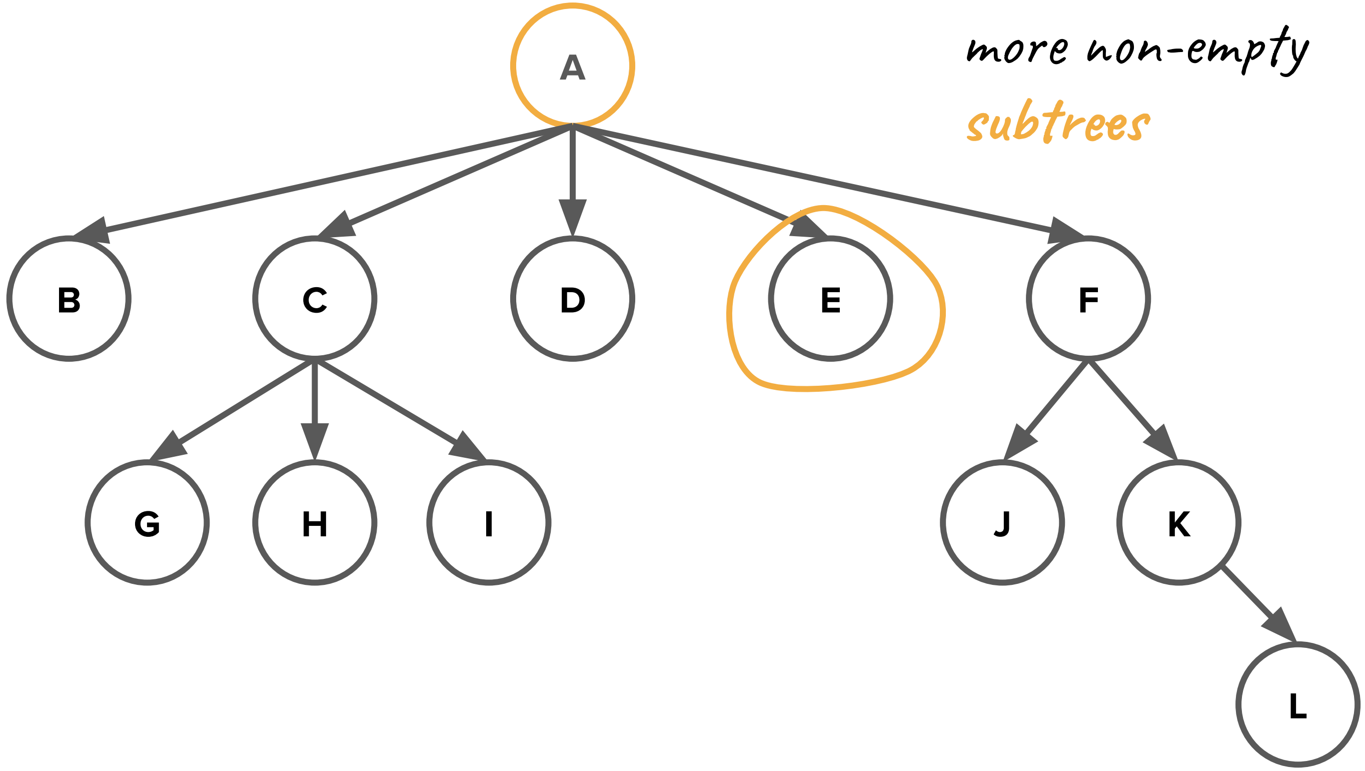

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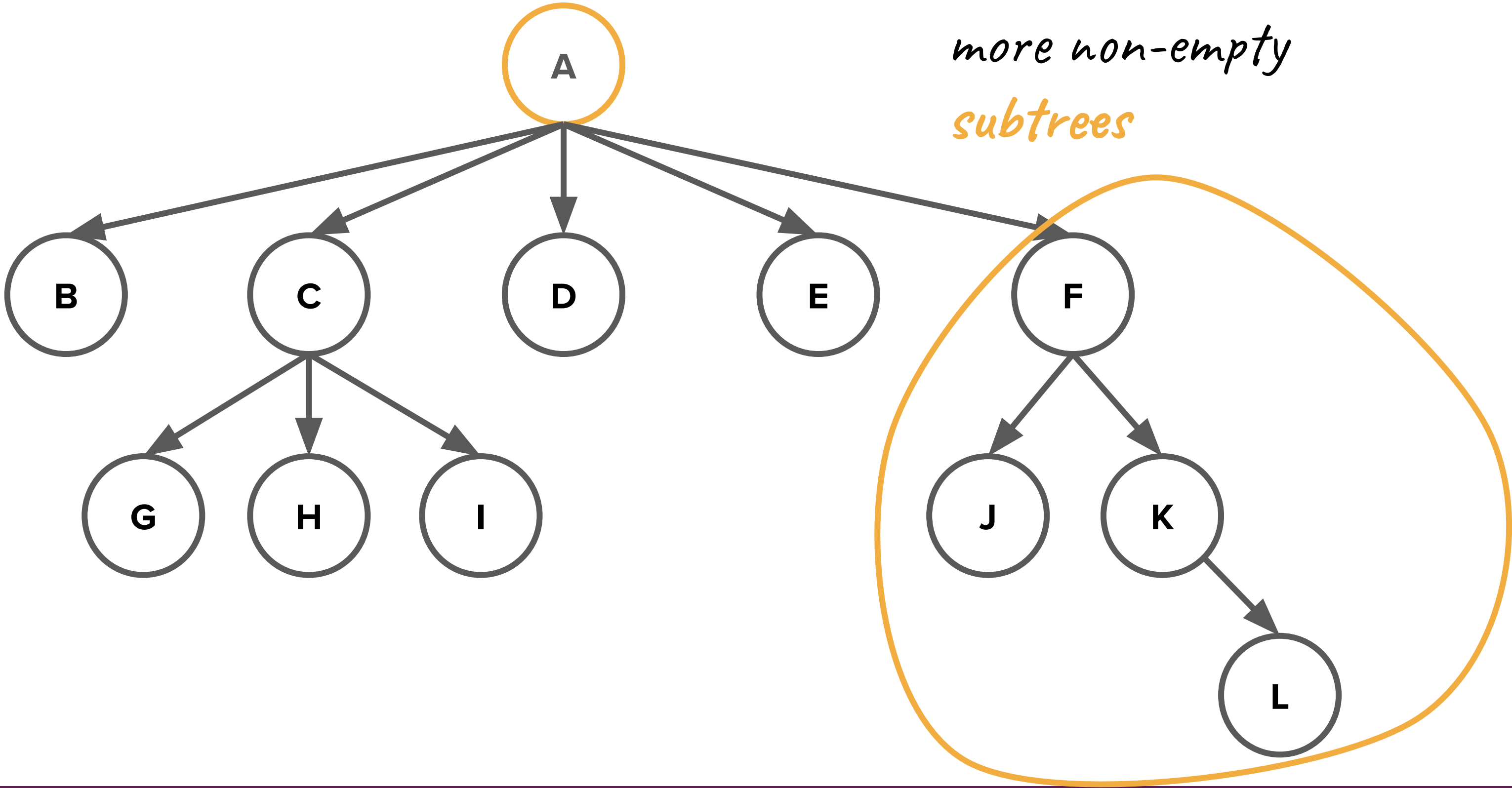

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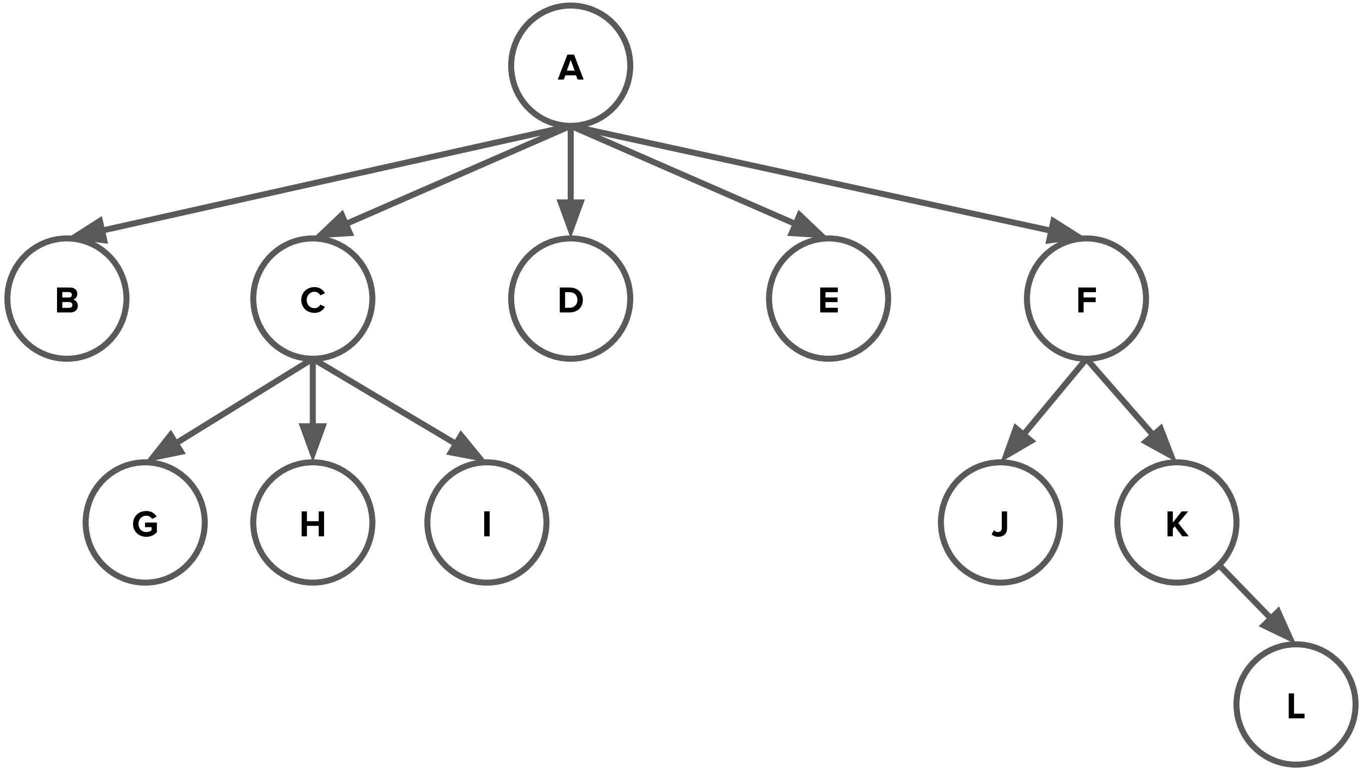

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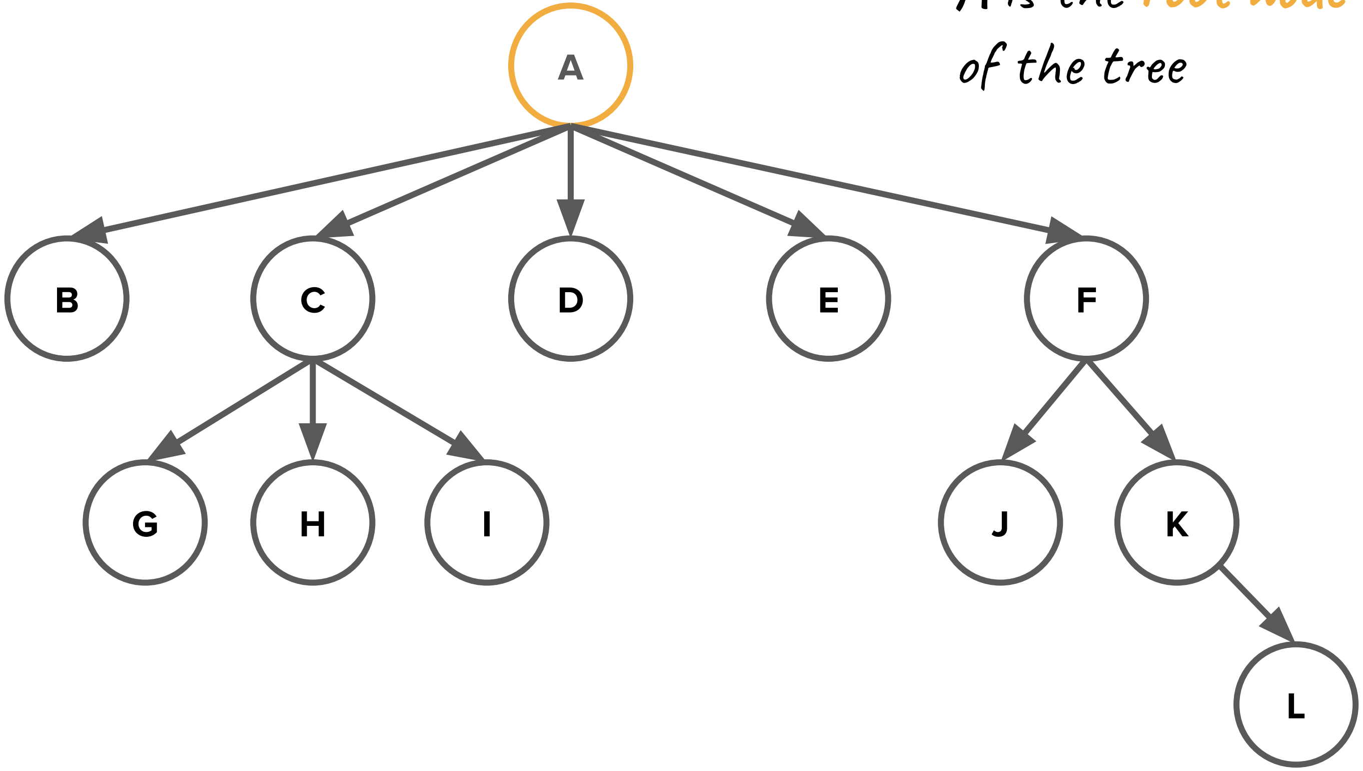
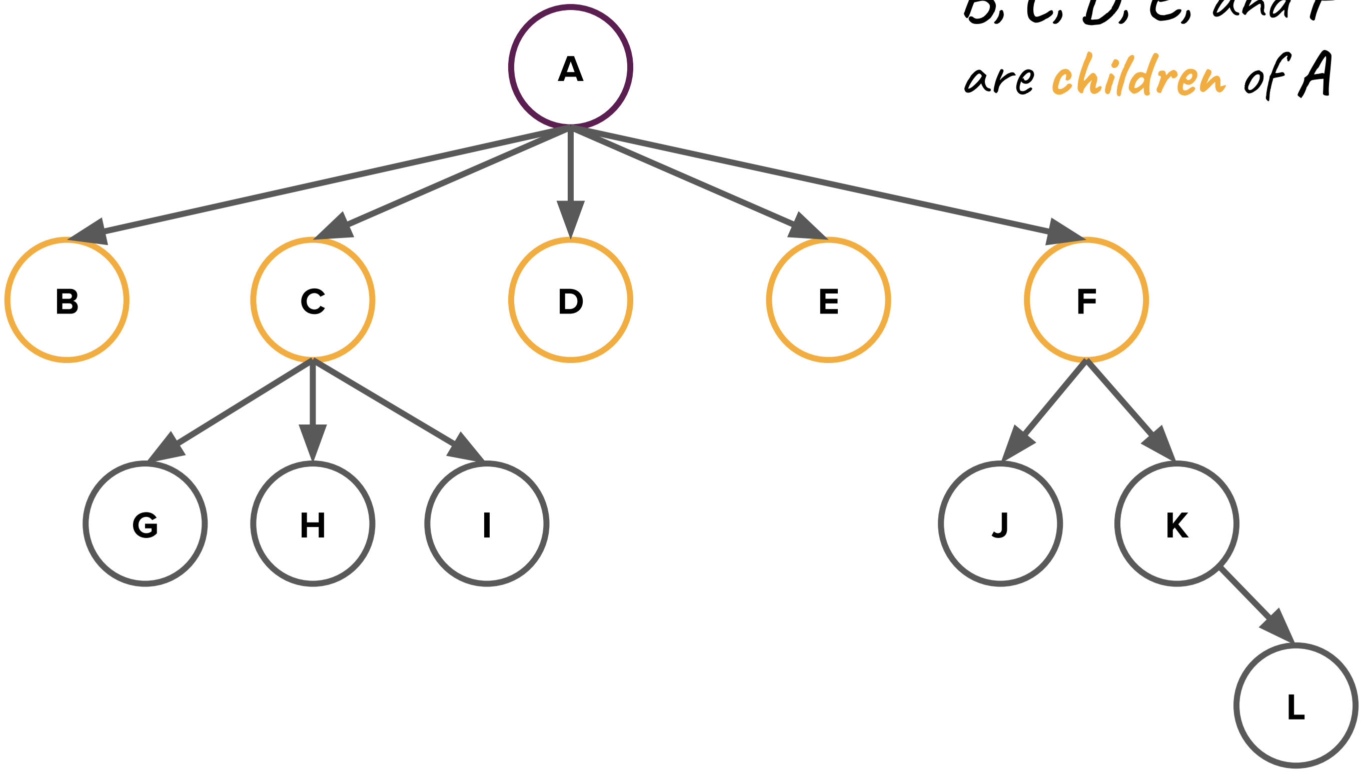


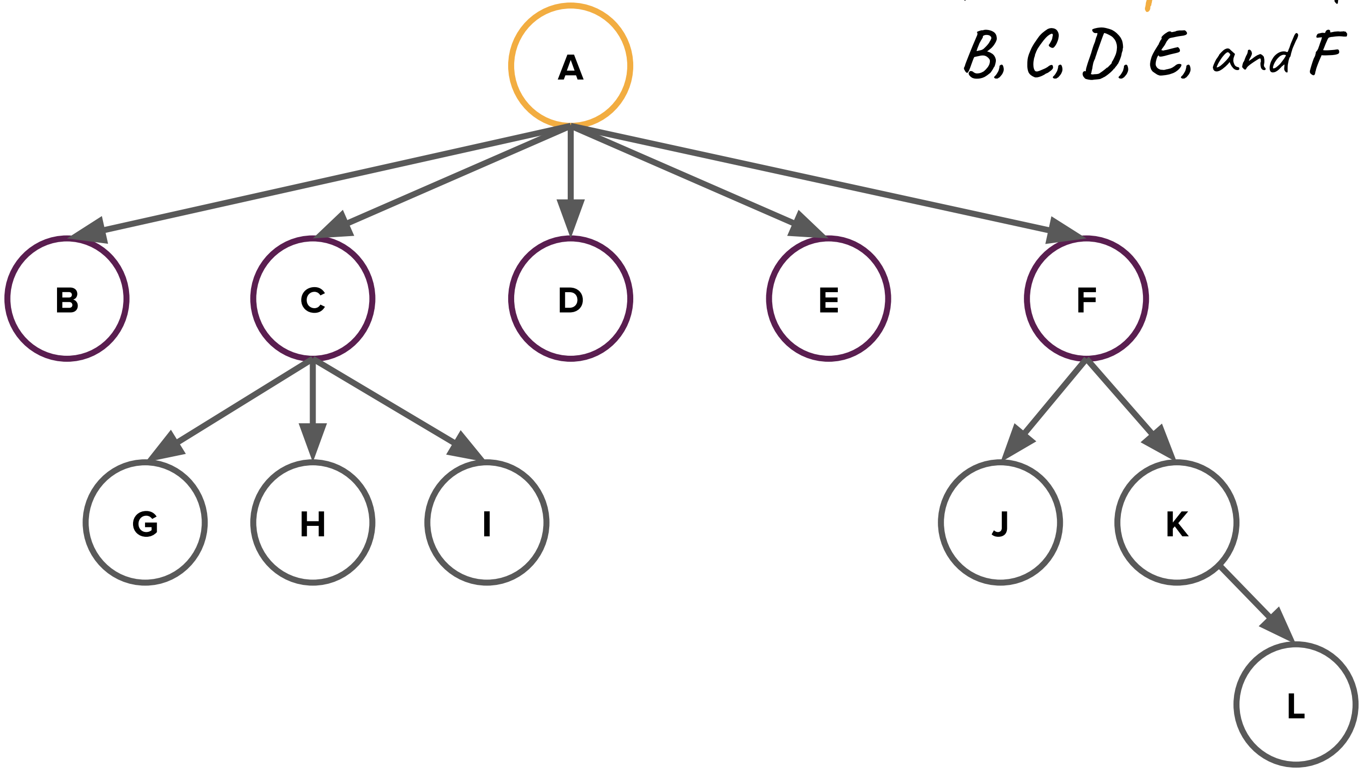
A is the root node of the tree

Tree Terminology



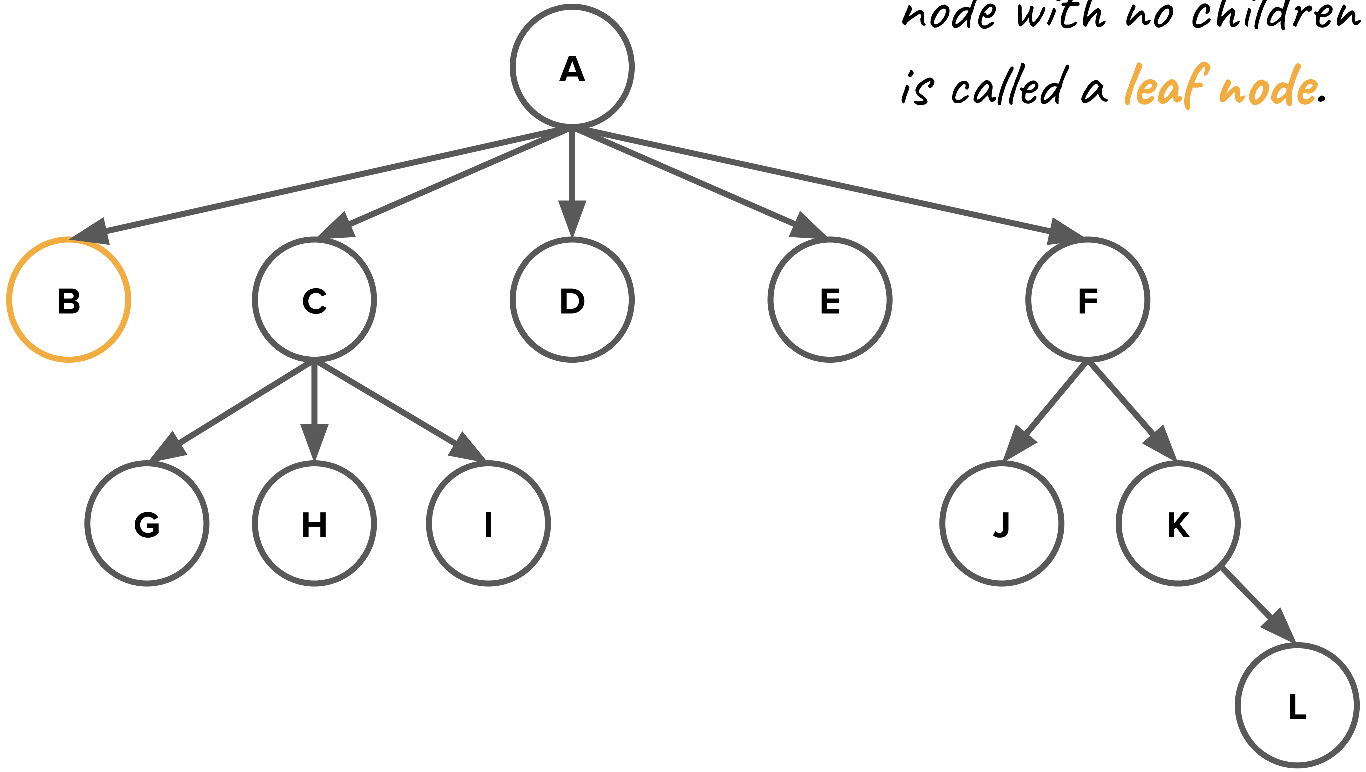
B, C, D, E, and F are *children* of A

Tree Terminology



A is the parent of B, C, D, E, and F

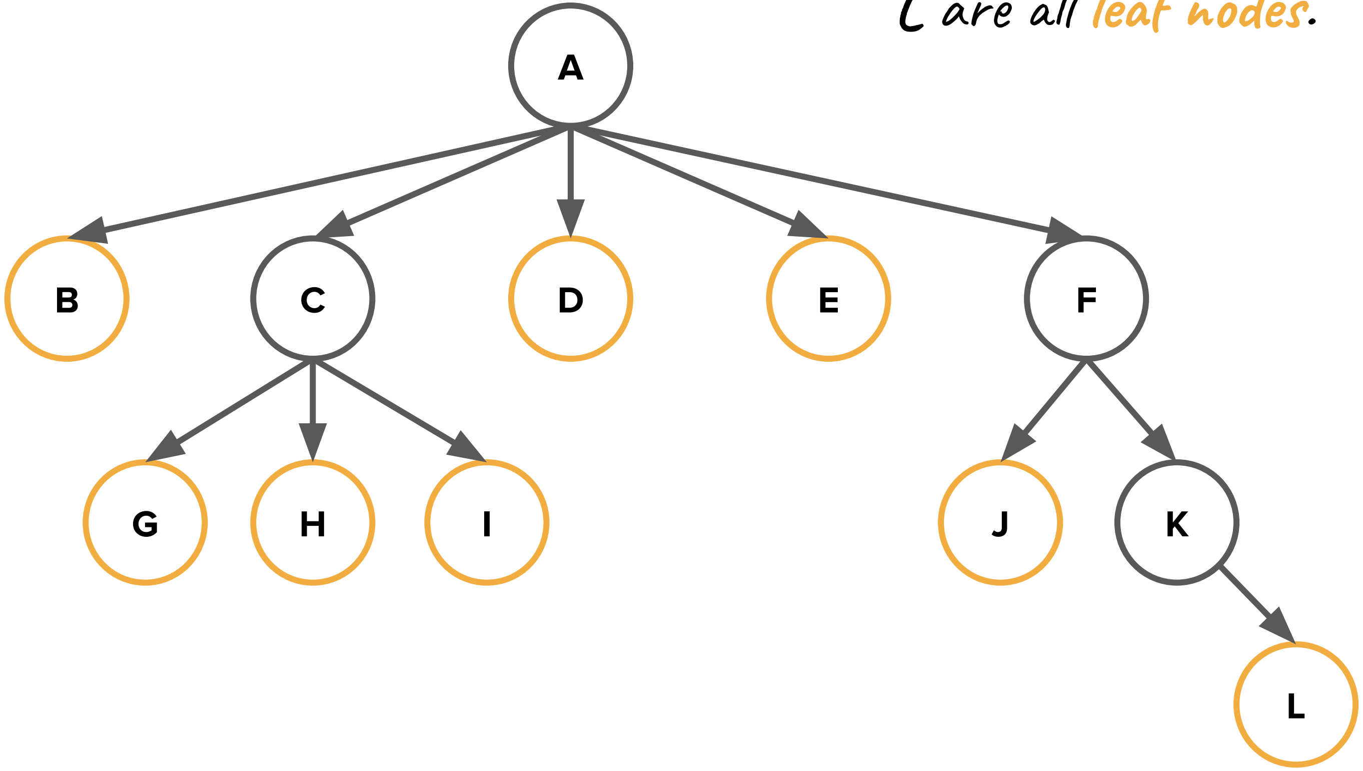
Tree Terminology



B has no children. A node with no children is called a leaf node.

Tree Terminology

B, G, H, I, D, E, J, and L are all *leaf nodes.*



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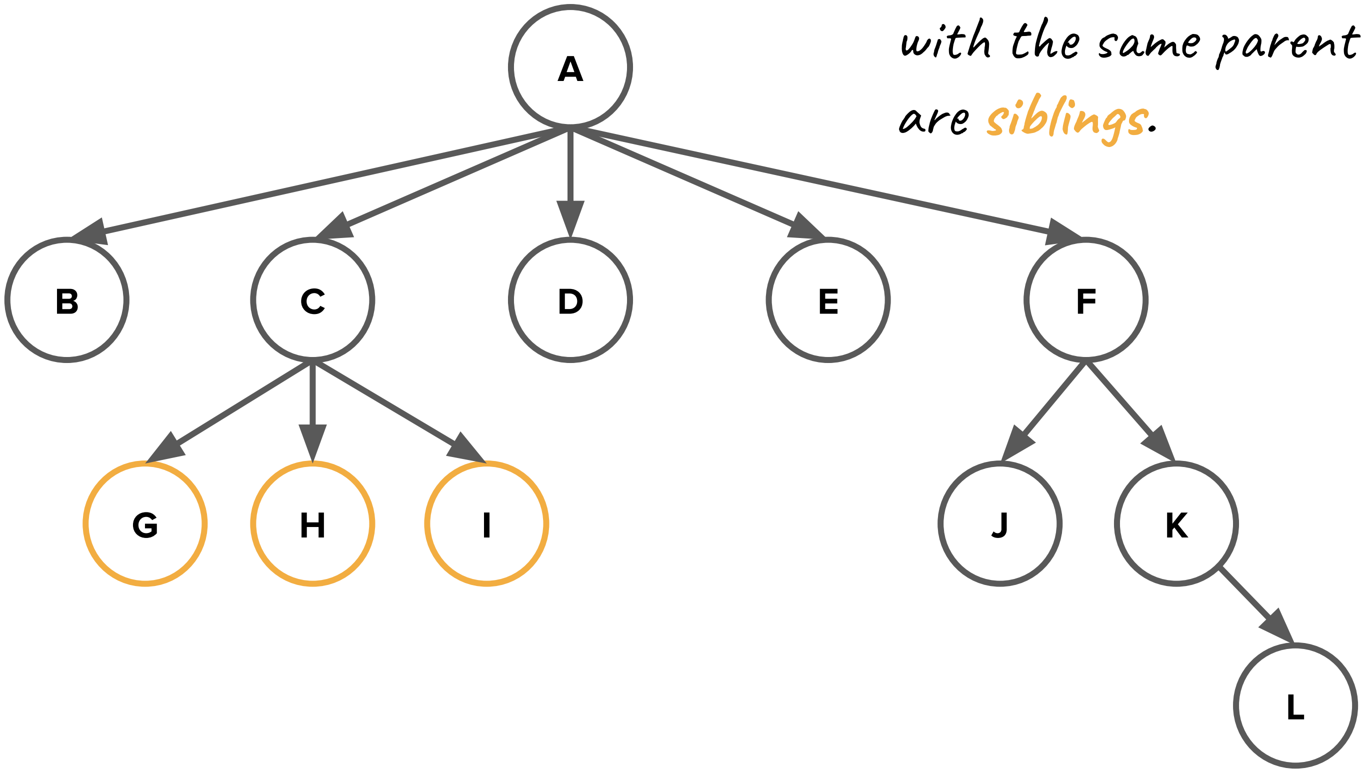
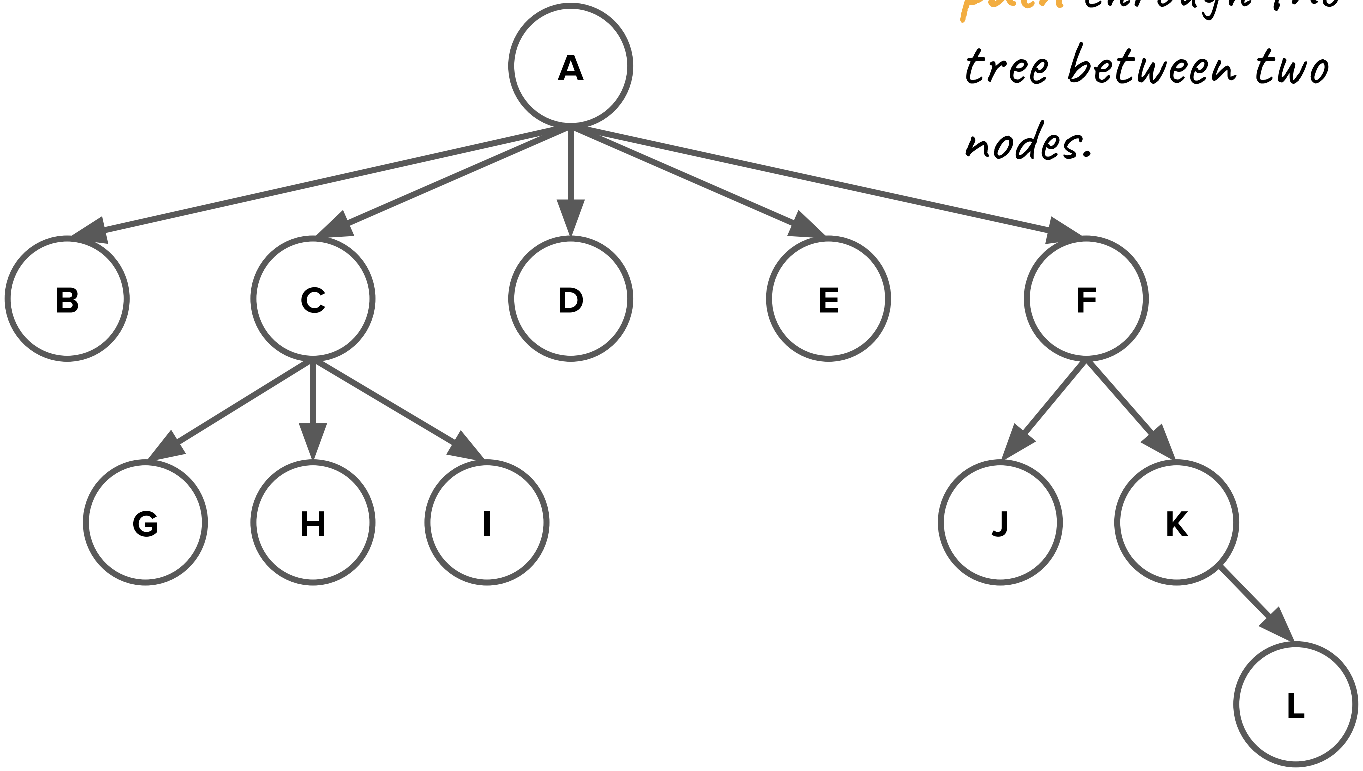


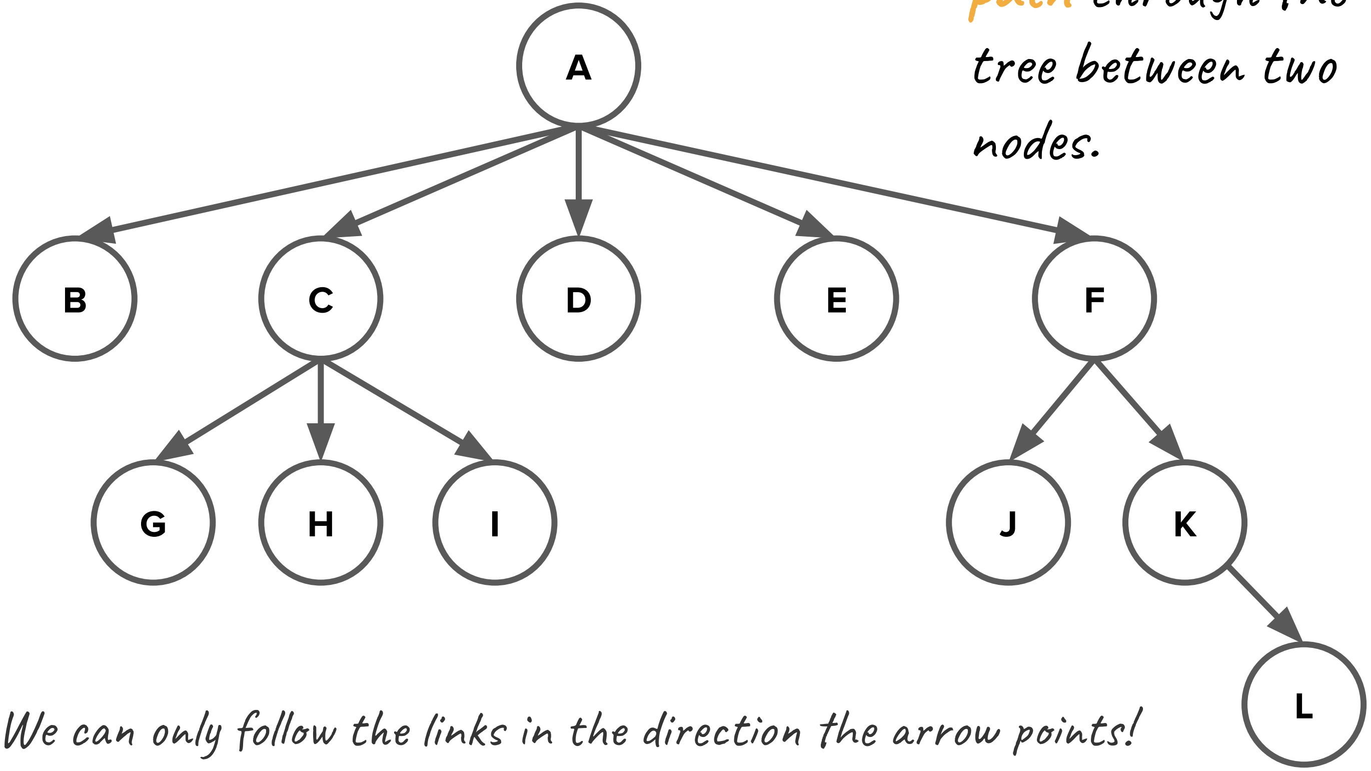
G, H and I all have the same parent. Nodes with the same parent are siblings.

Tree Terminology



We can define a **path** through the tree between two nodes.

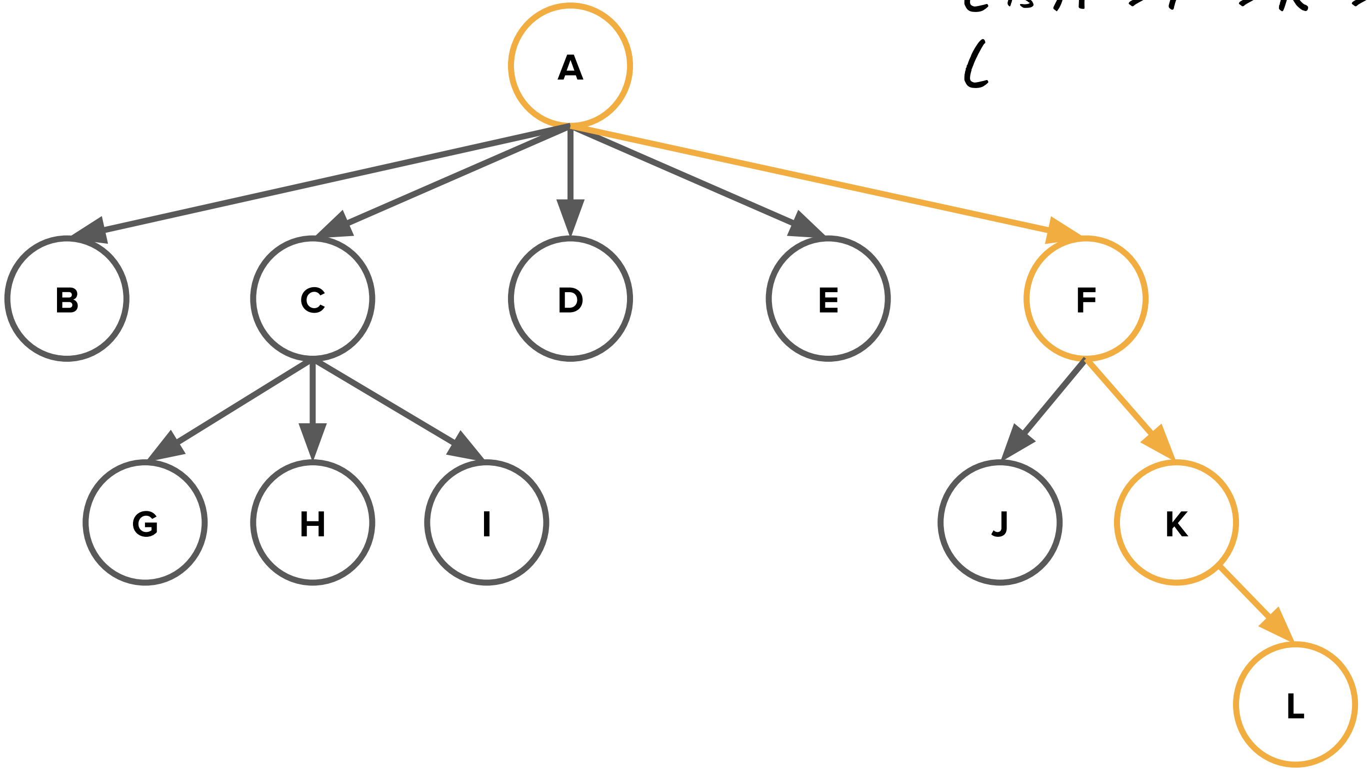
Tree Terminology



We can define a path through the tree between two nodes.

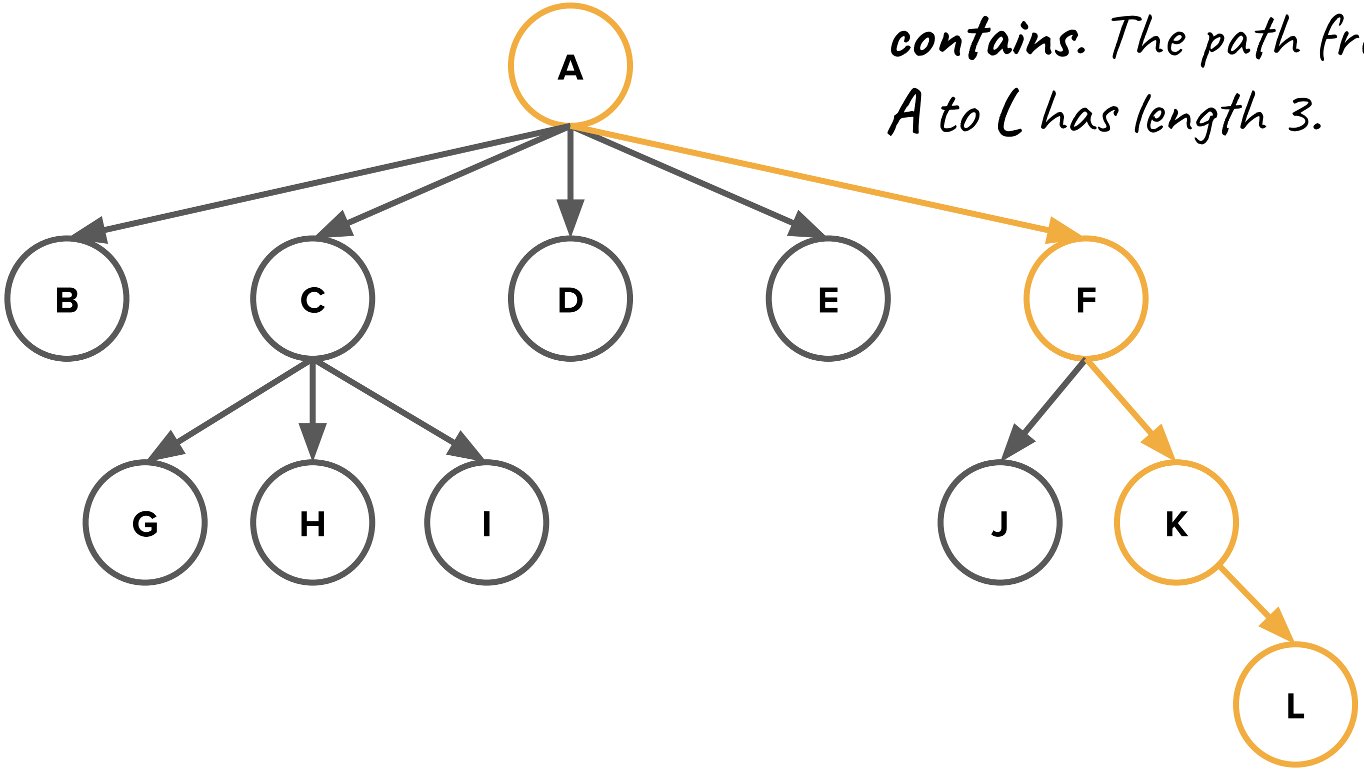
Note: We can only follow the links in the direction the arrow points!

Tree Terminology



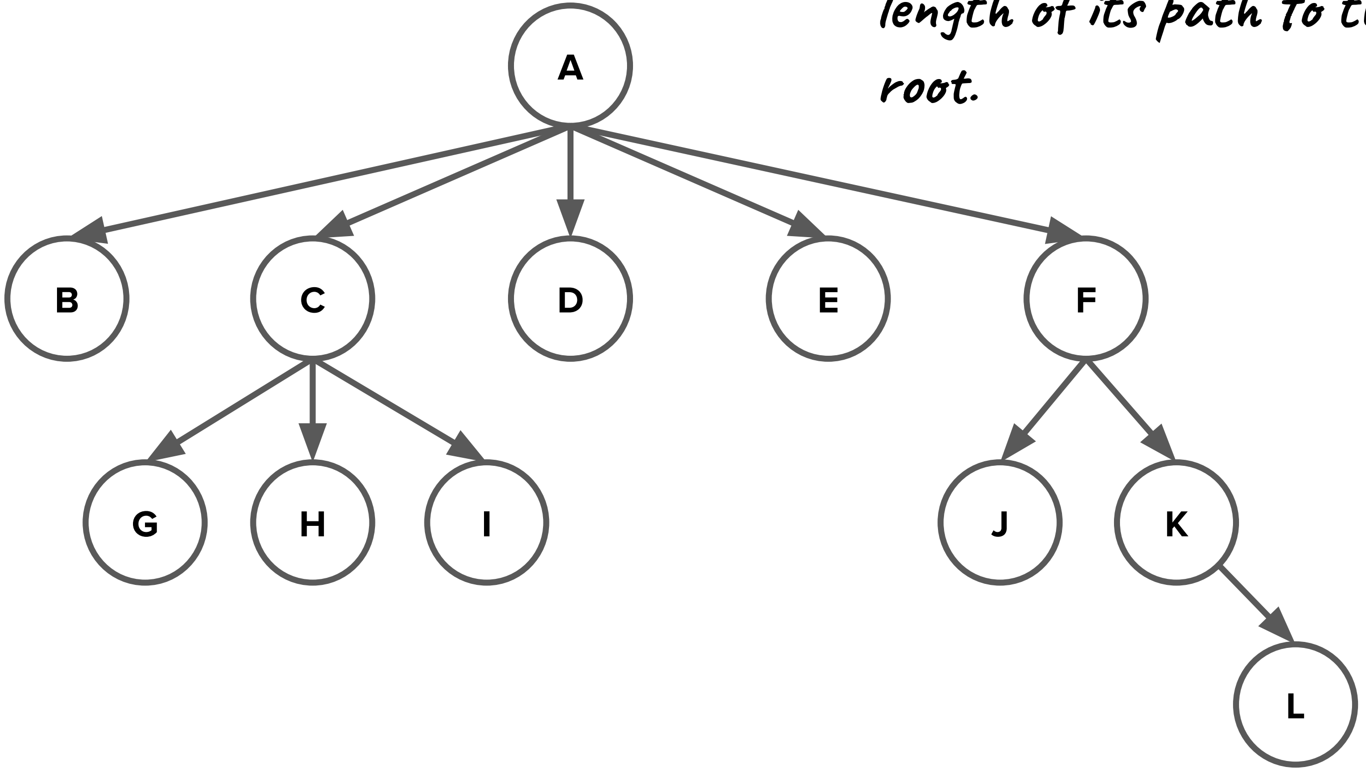
The path from A to L is A -> F -> K -> L

Tree Terminology



The length of the path is **number of edges it contains**. The path from A to L has length 3.

Tree Terminology



The depth of a node is the length of its path to the root.

Tree Terminology

depth: 0

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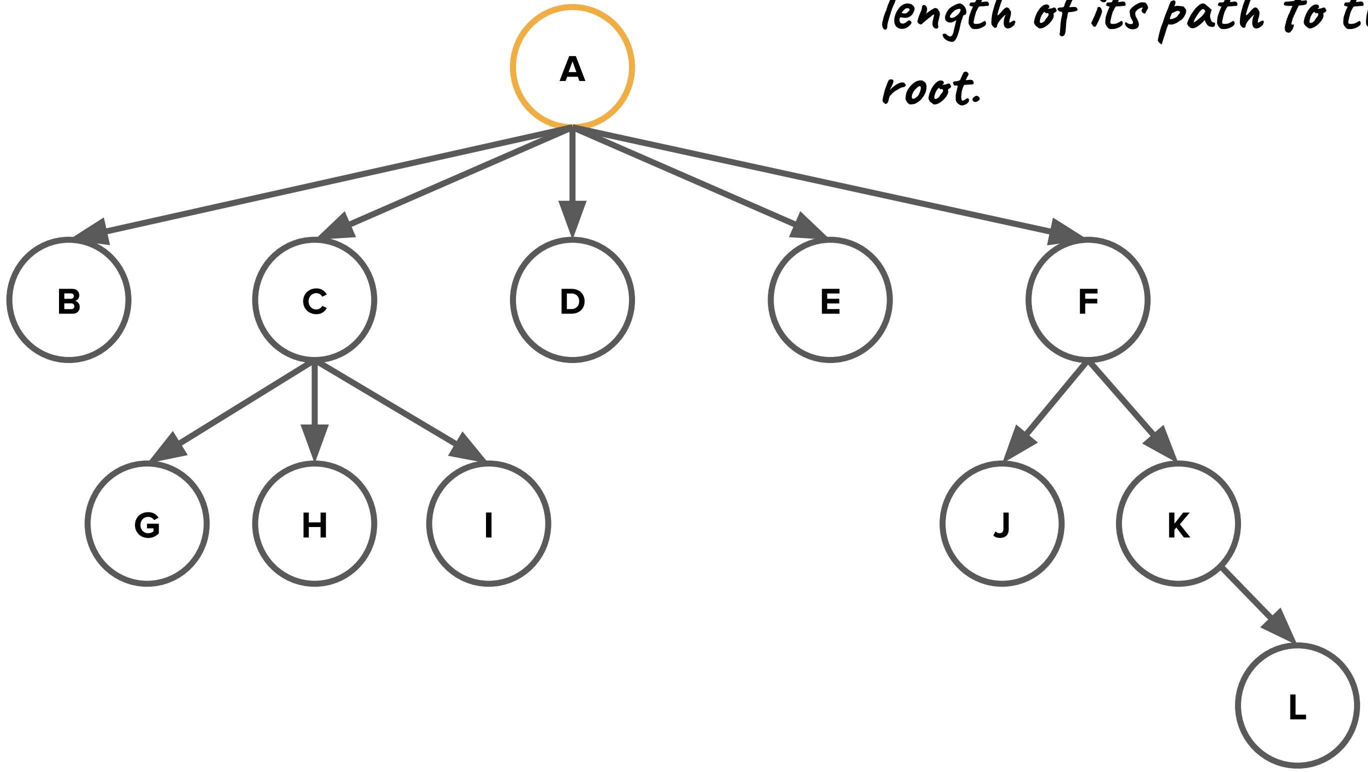

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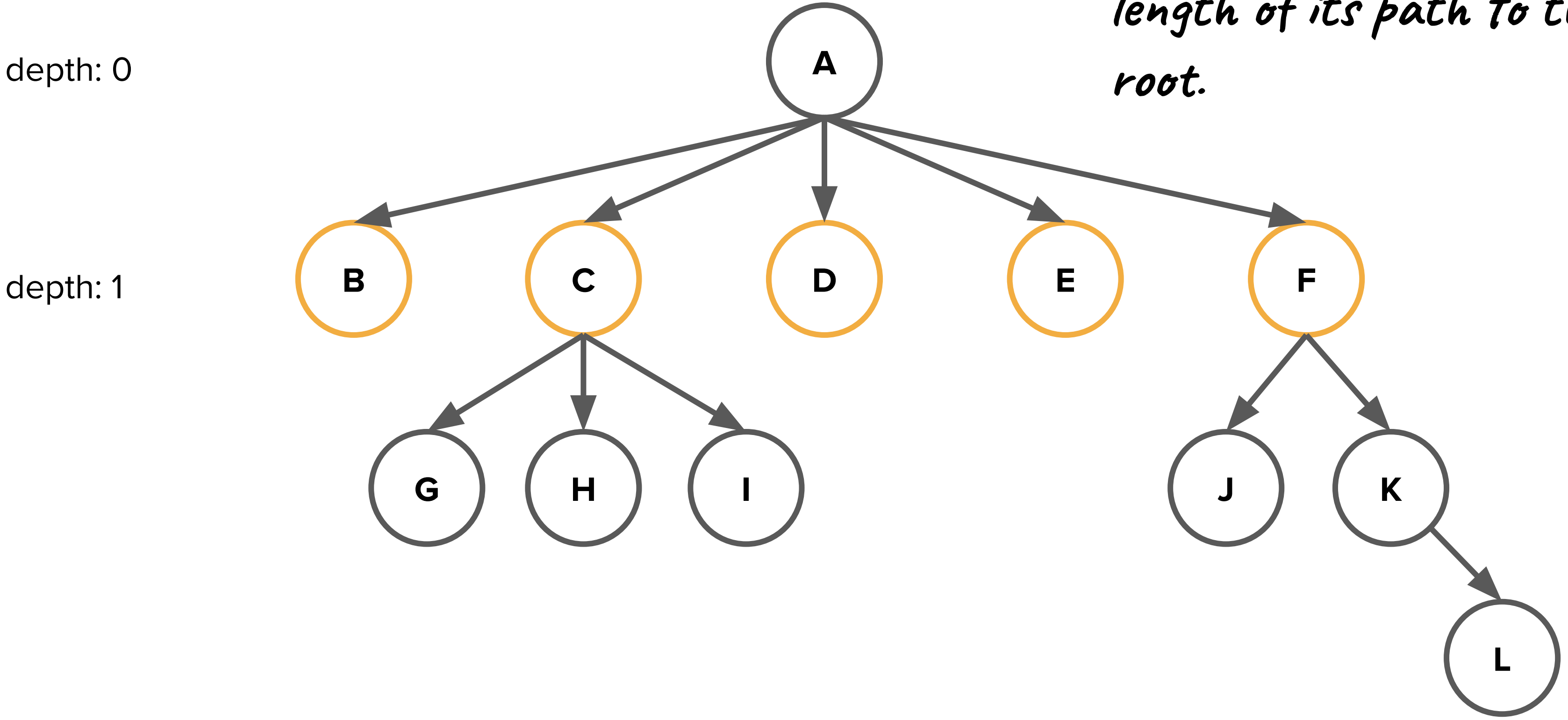

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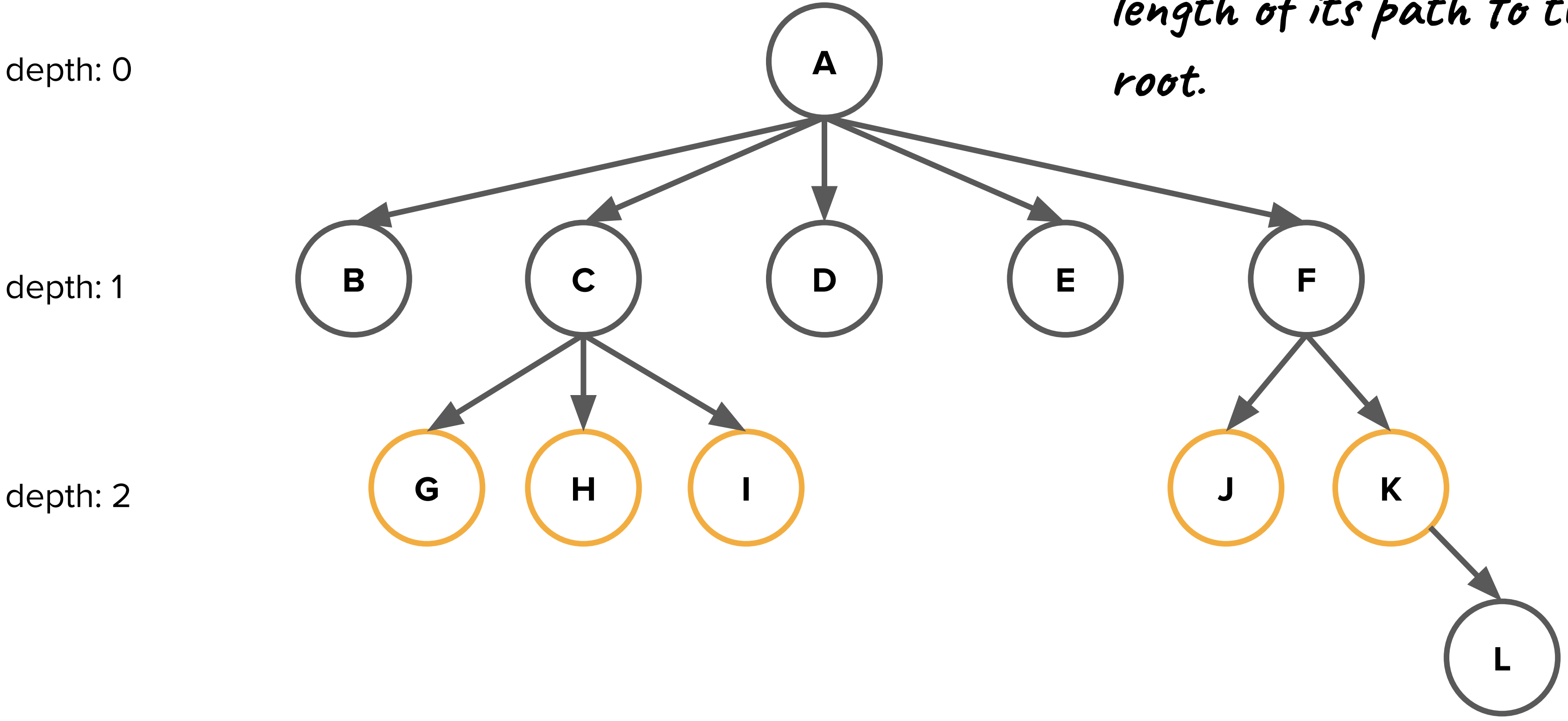

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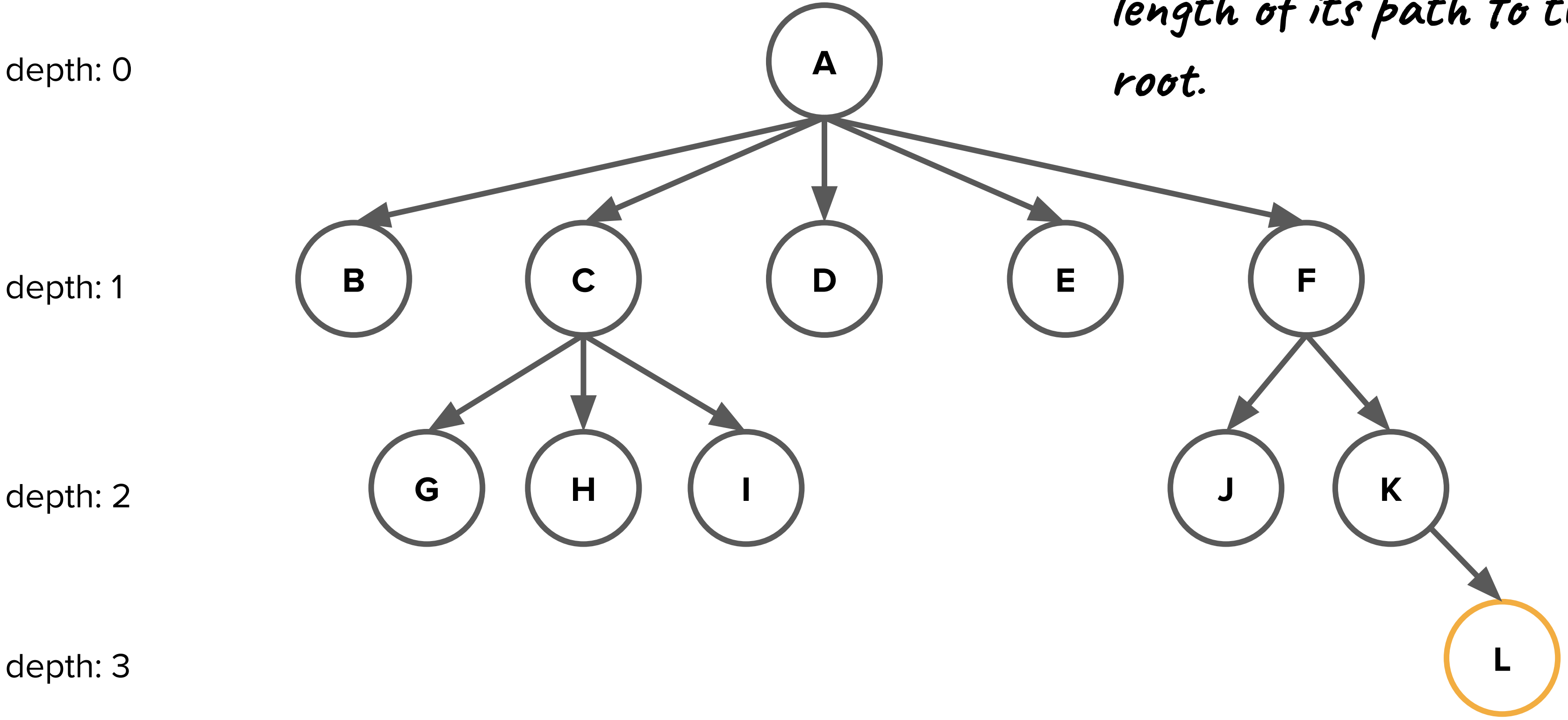
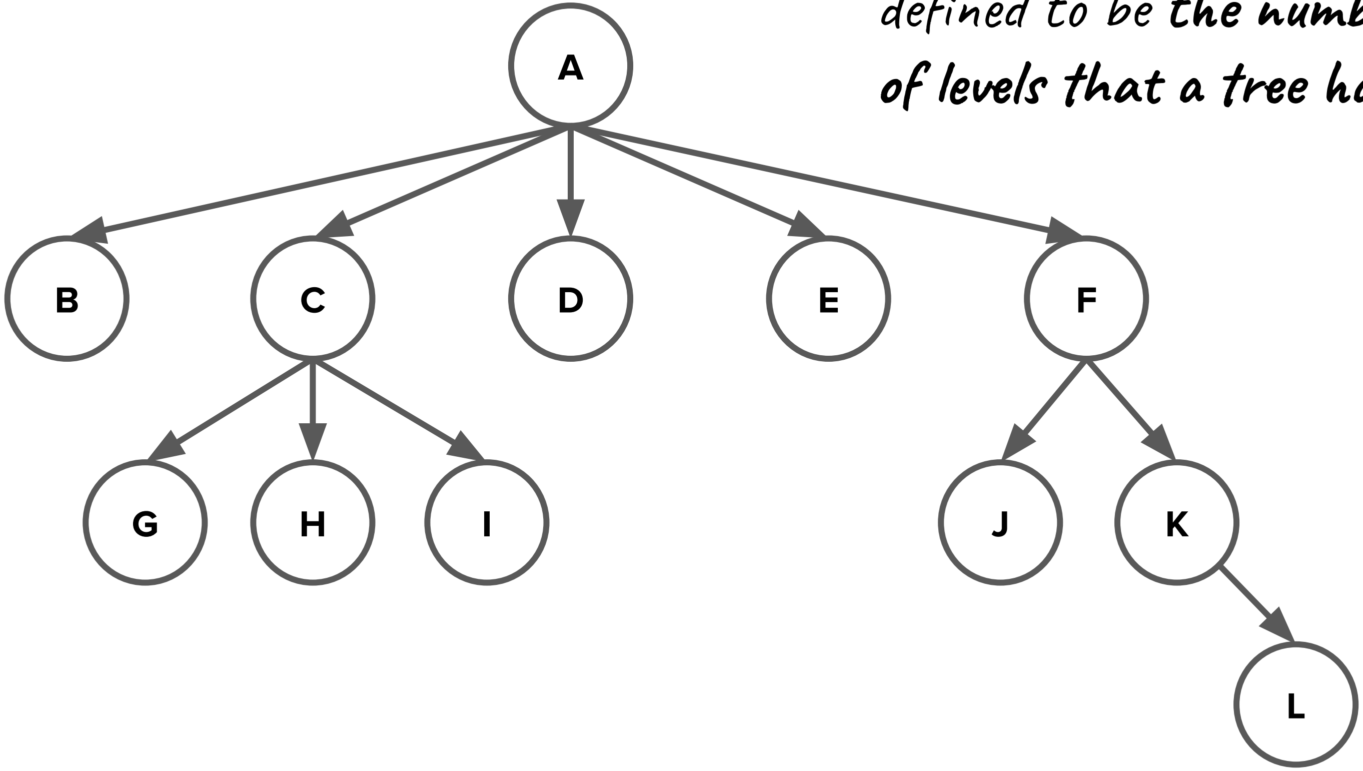


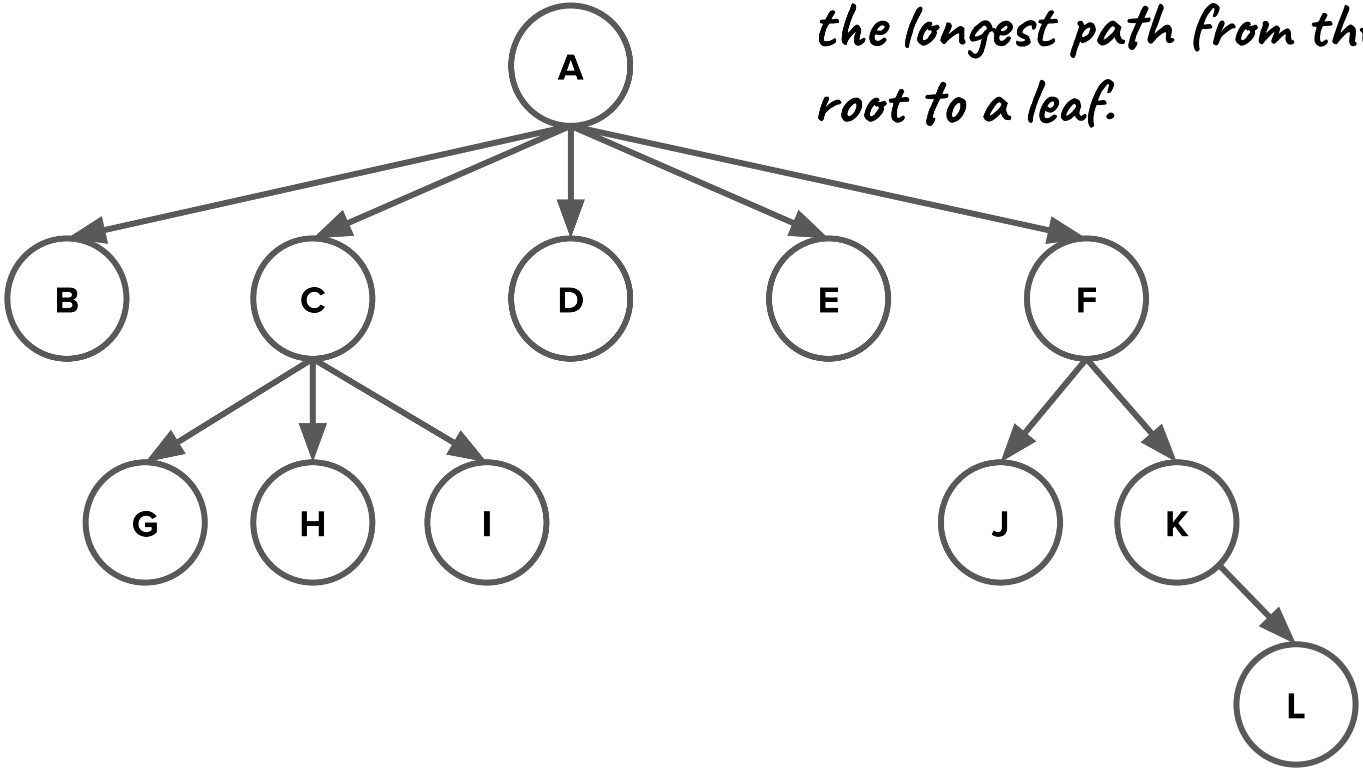
Tree Terminology



The *height* of a tree is defined to be **the number of levels that a tree has.**

Tree Terminology

The height can also be defined as **the number of nodes along the longest path from the root to a leaf.**



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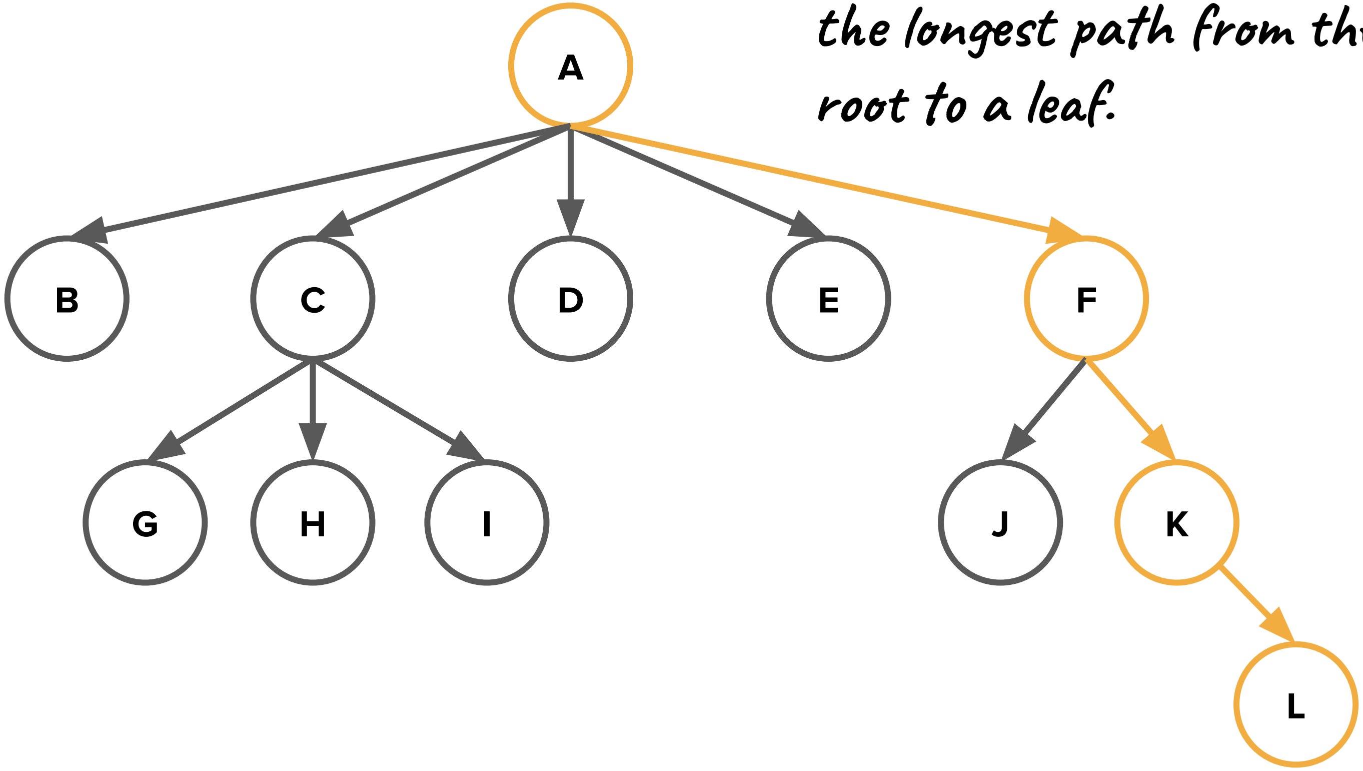

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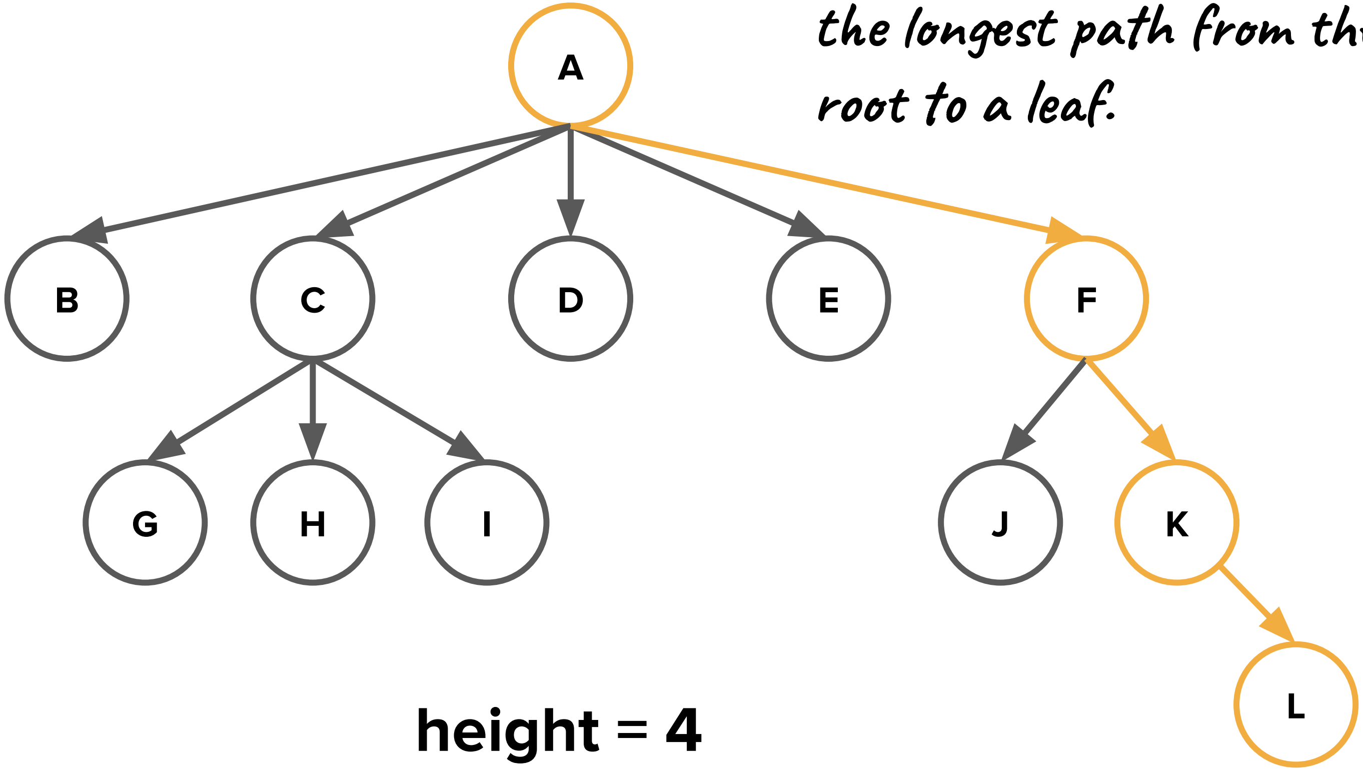

height = 4

Tree Terminology Summary

- Every non-empty tree has a **root node** that defines the "top" of the tree.
- Every node has 0 or more **children** nodes descended from it. Nodes with no children are called **leaf nodes**.
- Every node in a tree has exactly one **parent** node (except for the root node).
- A **path** through the tree traverses edges between parents and their children.
- The **depth** of a node is the length of the path between the root and that node. A tree's **height** is the number of nodes in the longest path through the tree.

Tree Properties

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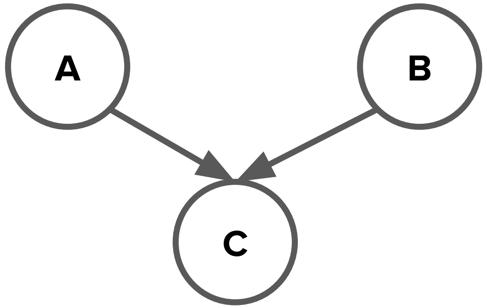

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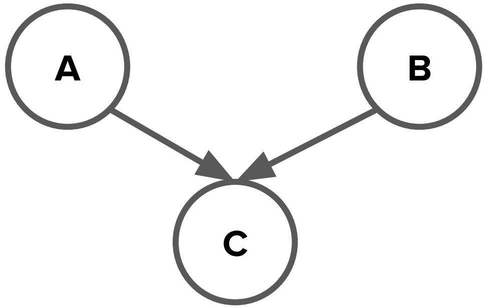

Not a tree!

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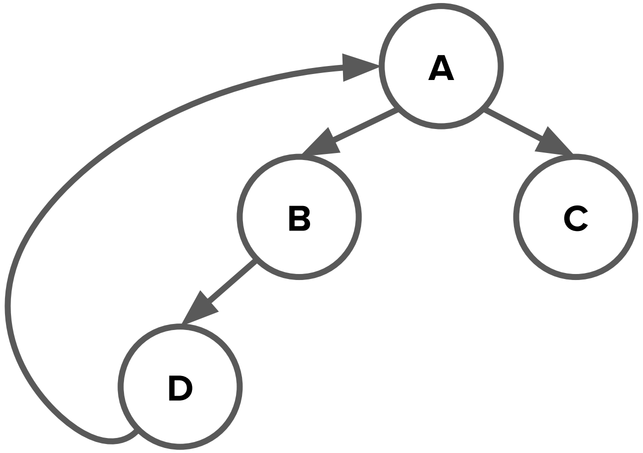

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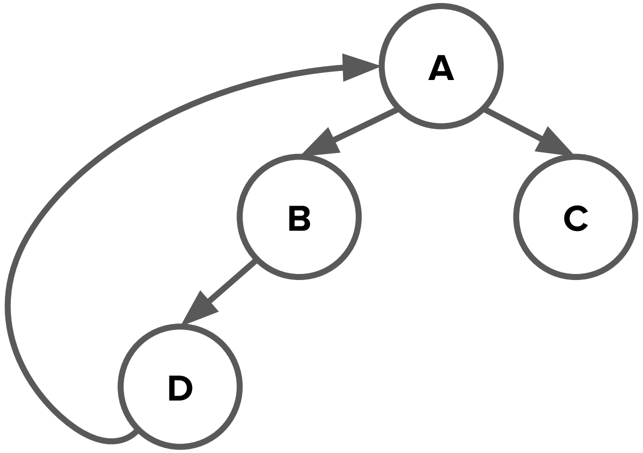
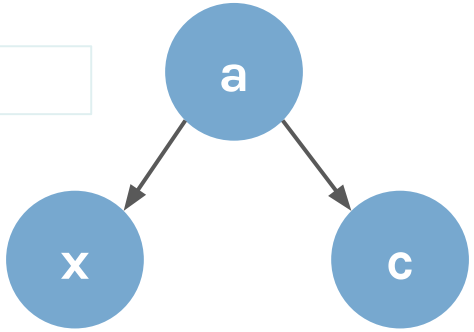


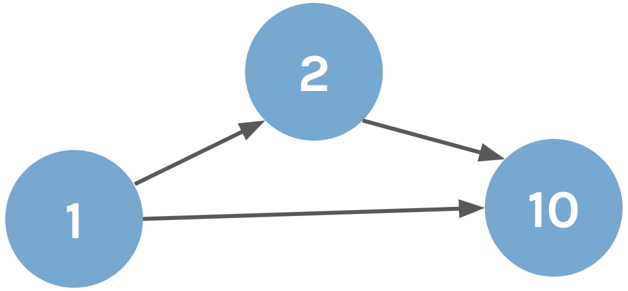
Not a tree!

Which of these are trees? pollev.com/cs106bpoll

A:



B:



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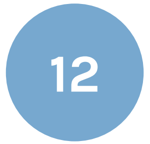
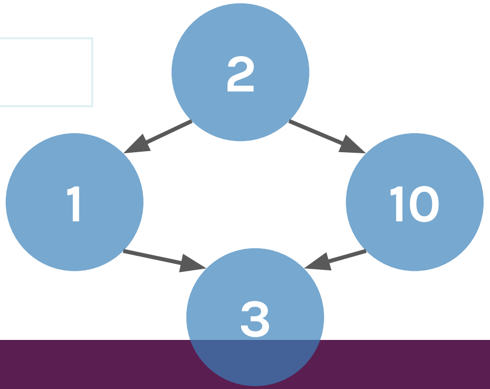
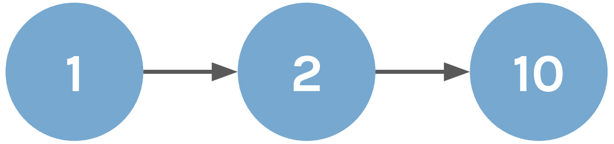


D:



E:



Announcements

Announcements

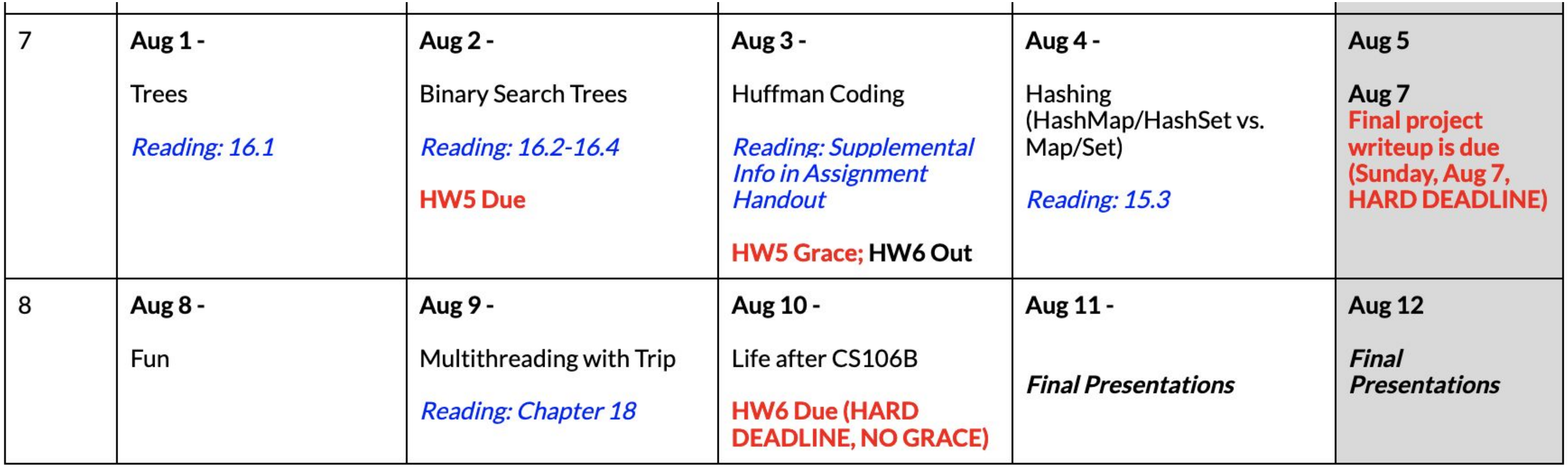
- Final project feedback was released this weekend!
  - Some of you may have received feedback requesting that you meet with one of us in order to receive full credit.
- Everyone is welcome to come to office hours this week
  - Trip’s Group OH on Friday, 8/5, from 10AM-12PM in Huang019
- Final project write-up due **THIS** Sunday, August 7. **No grace period.**
- **General feedback:**
  - Very creative ideas!
  - Don’t overscope (aka don’t bite off more than you can chew)

Announcements

- Assignment 5 is due tomorrow at 11:59 pm (with 24 hour grace period).
- Assignment 4 revisions due this Friday at 11:59 pm.
- Assignment 6 comes out Wednesday!
- Due to the end of quarter timeline, there will be **no revisions on Assignments 6**.

Announcements

- We’re so close!



7	**Aug 1 -** Trees *Reading: 16.1*	**Aug 2 -** Binary Search Trees *Reading: 16.2-16.4* **HW5 Due**	**Aug 3 -** Huffman Coding *Reading: Supplemental Info in Assignment Handout* **HW5 Grace; HW6 Out**	**Aug 4 -** Hashing (HashMap/HashSet vs. Map/Set) *Reading: 15.3*	**Aug 5** **Aug 7** **Final project writeup is due (Sunday, Aug 7, HARD DEADLINE)**
8	**Aug 8 -** Fun	**Aug 9 -** Multithreading with Trip *Reading: Chapter 18*	**Aug 10 -** Life after CS106B **HW6 Due (HARD DEADLINE, NO GRACE)**	**Aug 11 -** ***Final Presentations***	**Aug 12** ***Final Presentations***

Trees in C++

Binary Trees

- In general, we've seen that nodes in a tree can have variable numbers of children (subtrees) and sometimes very, very many.

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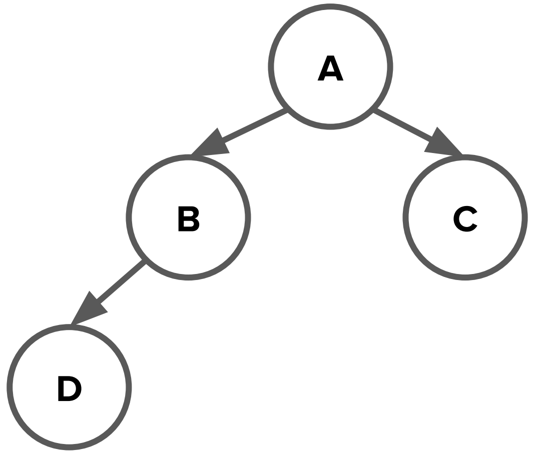
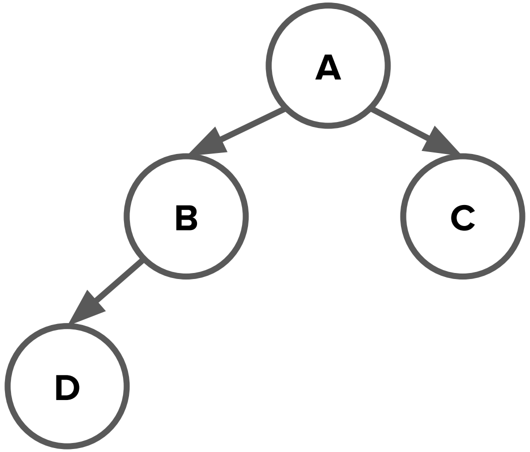
- However, when working with trees in computer programs, it is common to work mostly with **binary trees**.
- A **binary tree** is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.
- Typically, the two children of a node in a binary tree are referred to as the **left child** and the **right child**.

Binary Trees

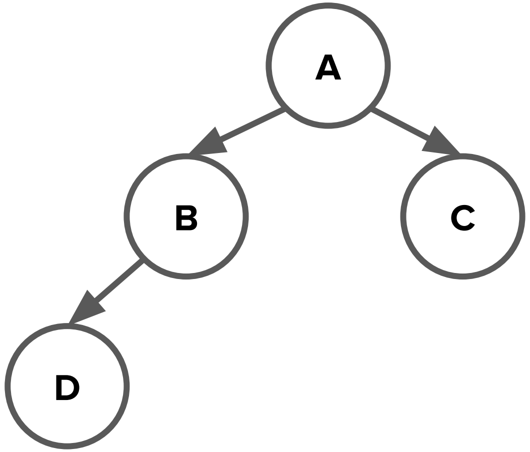


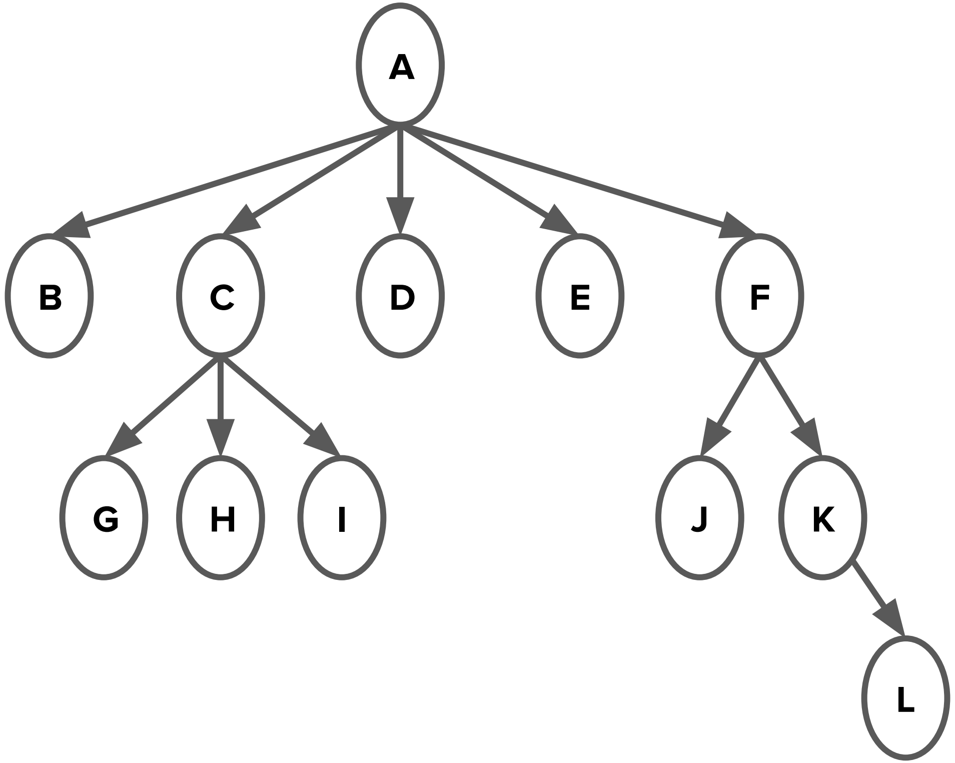
Binary Trees



Binary Tree!

Binary Trees





Binary Tree!

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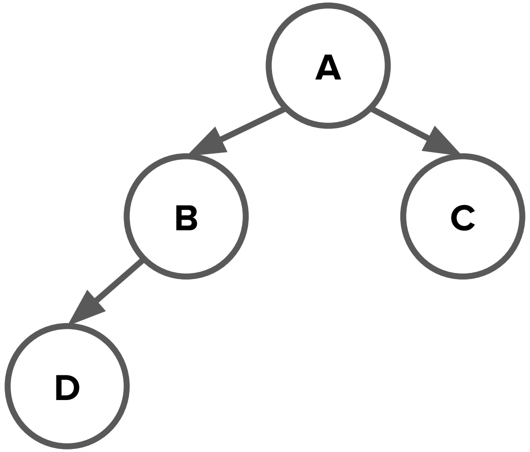
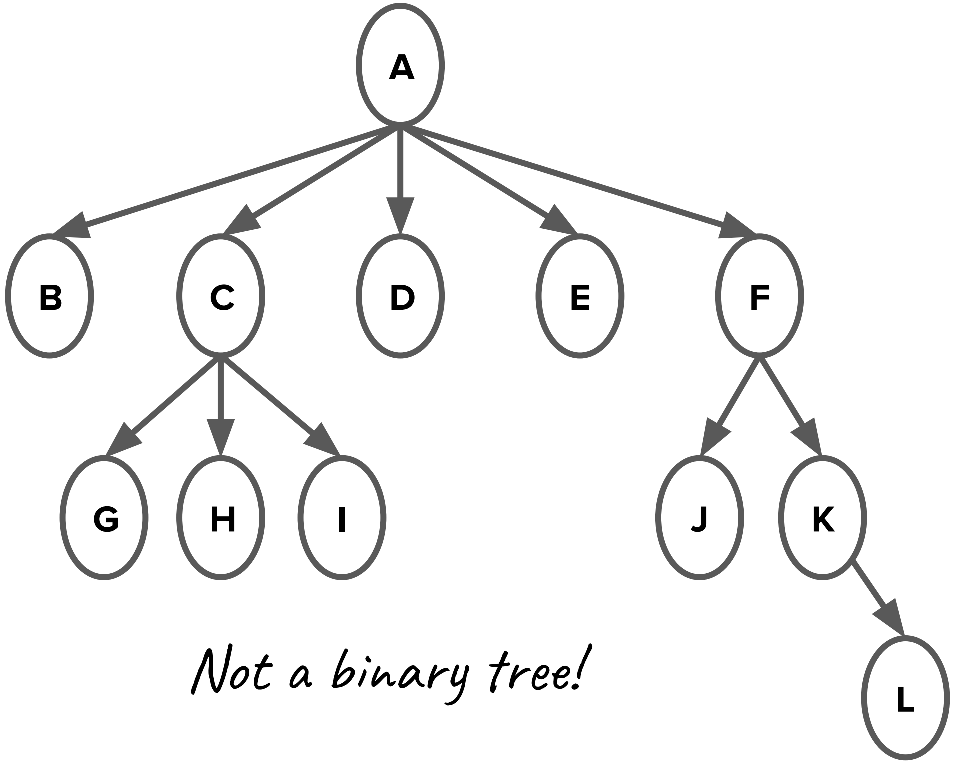


Binary Tree!



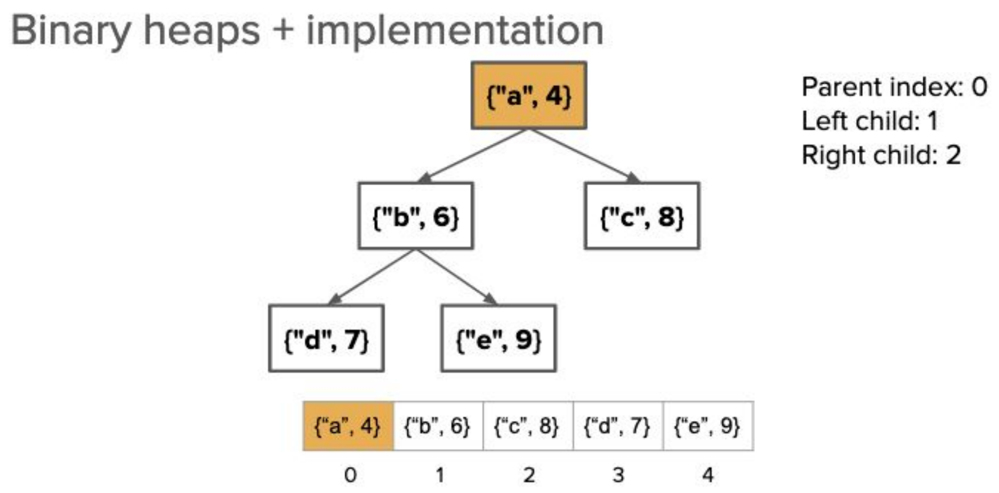
Not a binary tree!

Building Trees Programmatically

- To build a tree in C++, we need a new version of the Node struct we've seen before.

Building Trees Programmatically

Wait… didn't we already build a binary tree in PQHeap?



Binary heaps + implementation

Parent index: 0
Left child: 1
Right child: 2

{"a", 4}	{"b", 6}	{"c", 8}	{"d", 7}	{"e", 9}
0	1	2	3	4

Building Trees Programmatically

- To build a tree in C++, we need a new version of the Node struct we've seen before.

- In this case, we want each Node to have a data value (like a linked list), but now we want two pointers, one to the left child, and one to the right child.

Building Trees Programmatically

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```
struct TreeNode {
    string data;
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```
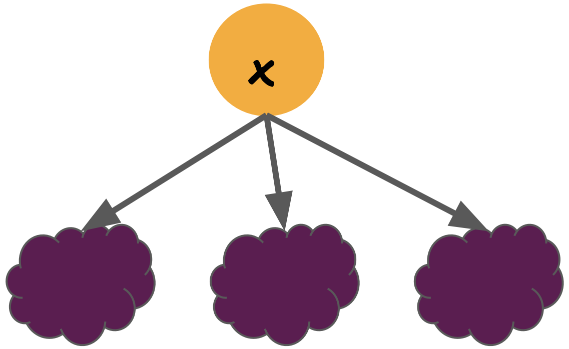

What is a tree?

A tree is either...

An empty data structure, or...

A single node (parent), with zero or more non-empty subtrees (children)



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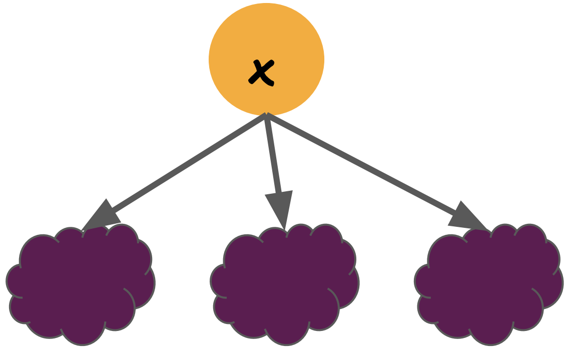

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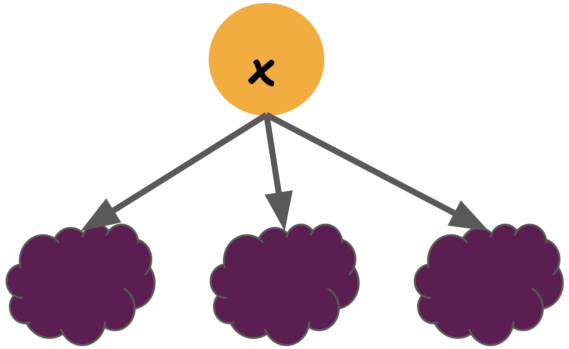
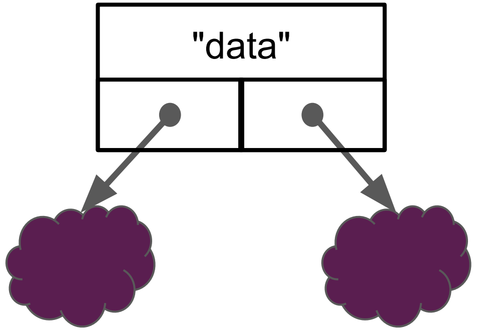


What is a tree in C++?

A tree is either...

An empty tree represented by **`nullptr`**, or...

A single **`TreeNode`**, with 0, 1, or 2 non-null pointers to other **`TreeNodes`**



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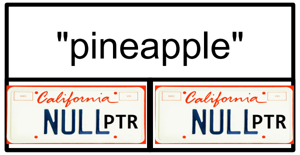

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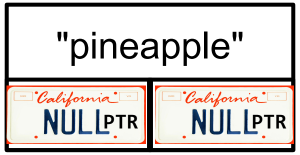

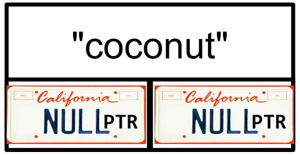

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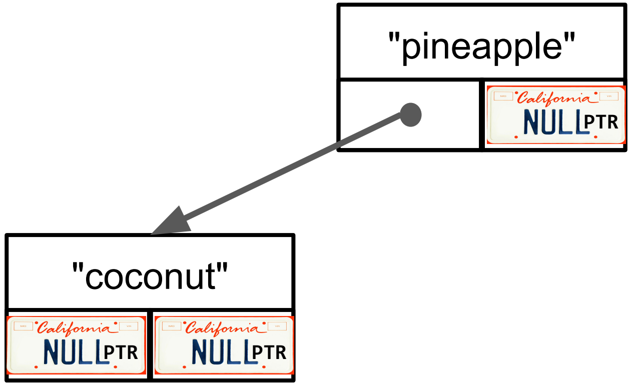

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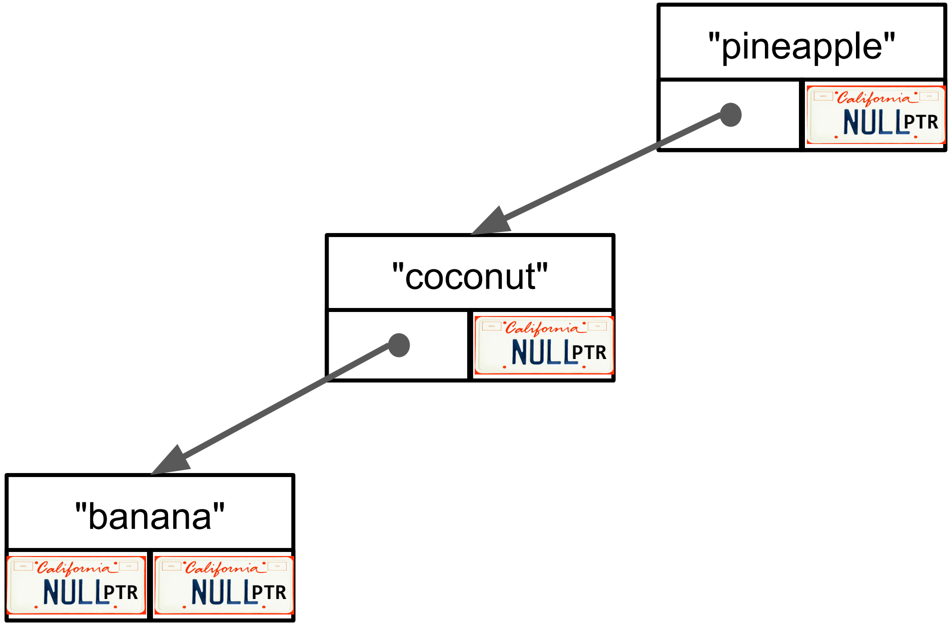

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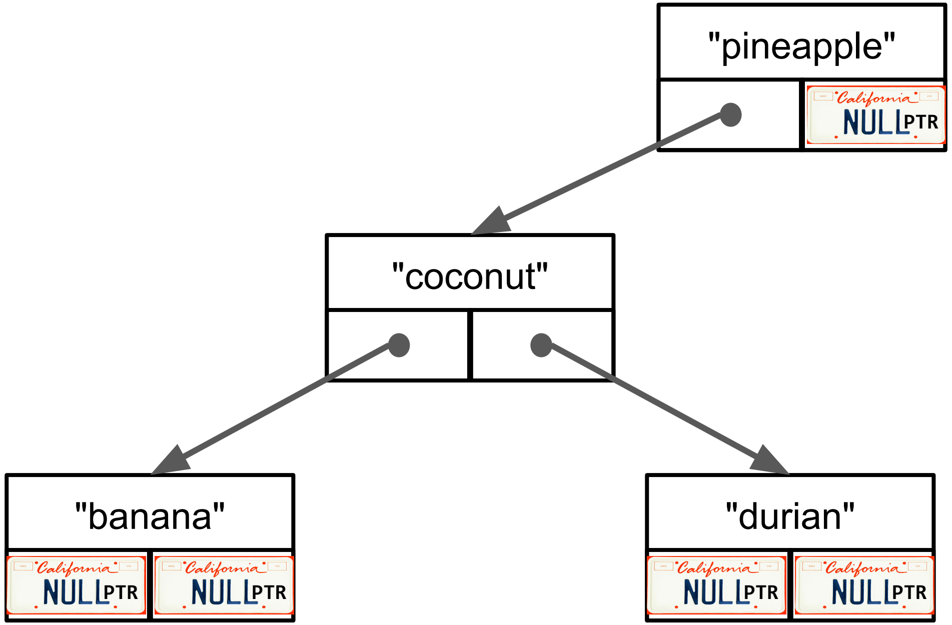

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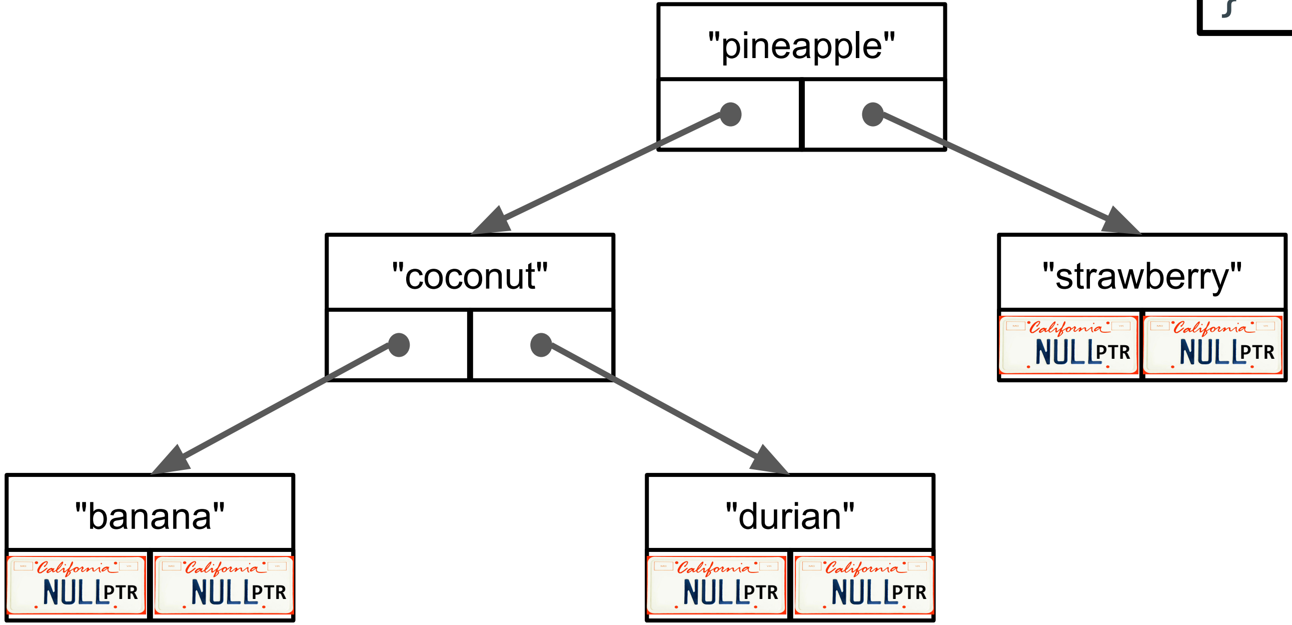

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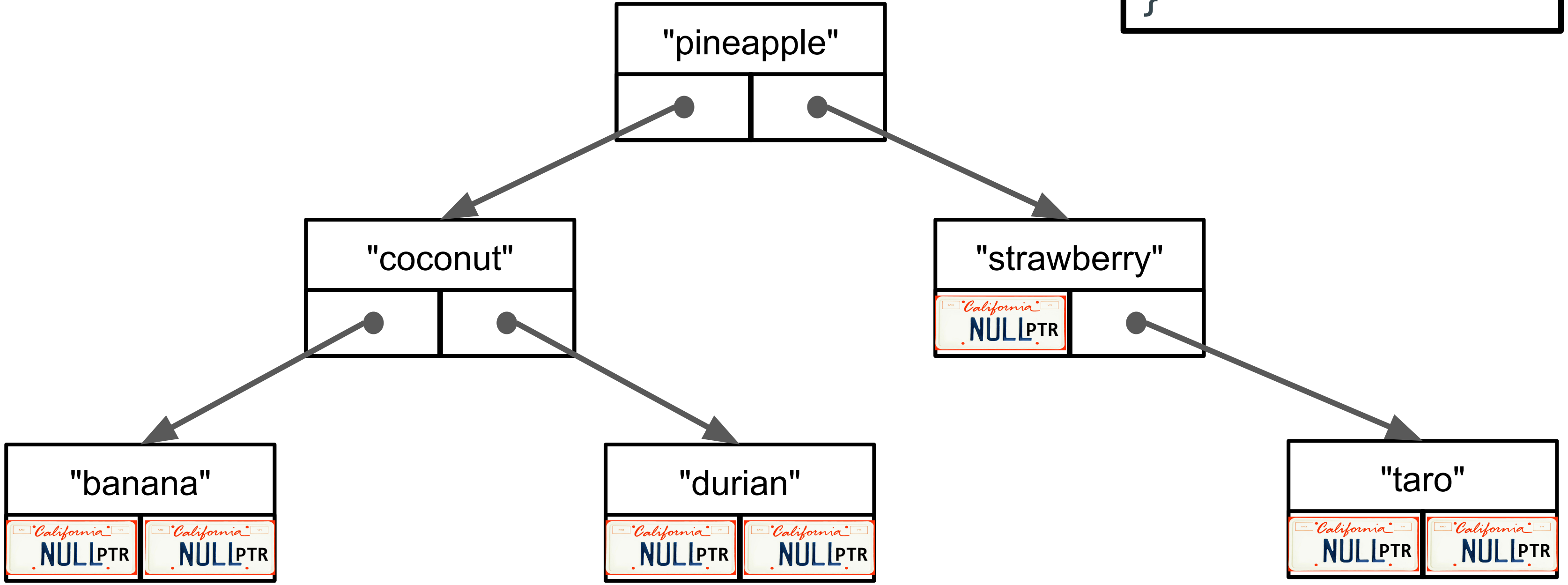

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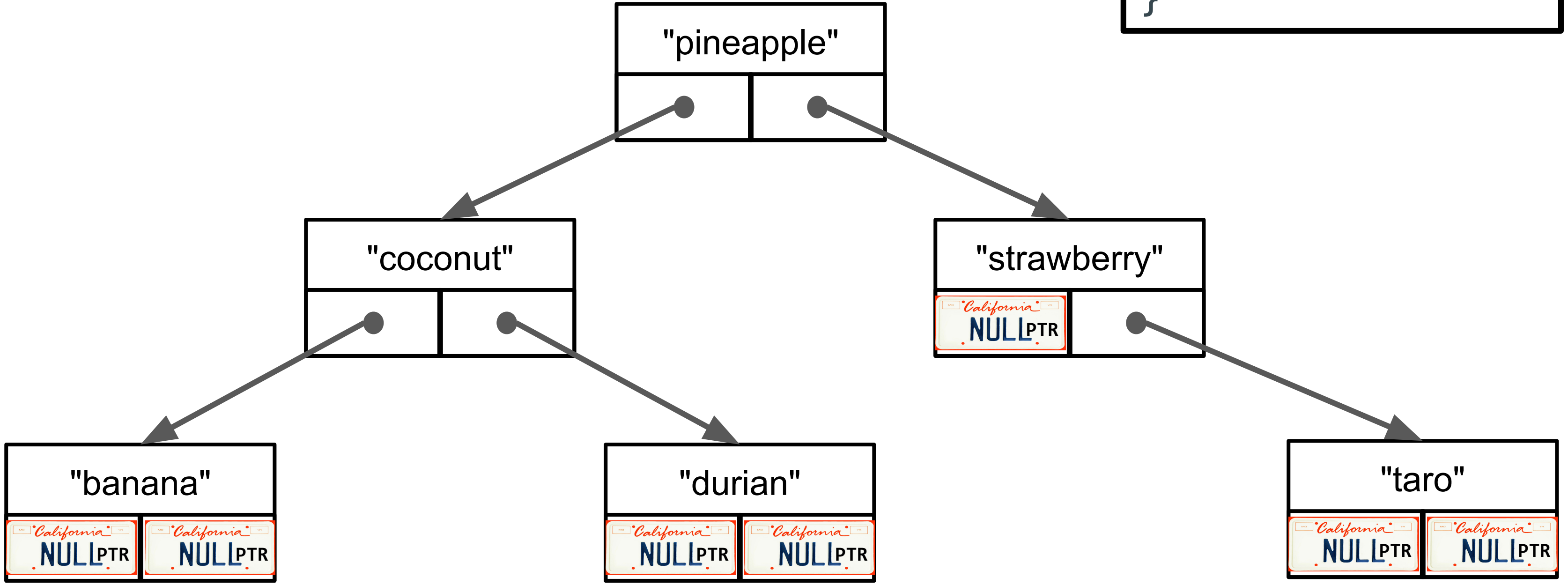

Note: Trees do not have to be complete, like heaps. Any node can have 0, 1, or 2 children.

Let's code it!

`buildExampleTree()`

Building a Tree Takeaways

- Building a tree is very similar to the process of building a linked list.
- We create new nodes of the tree by dynamically allocating memory.
- We integrate these new nodes into the tree by rewiring the **`left`** and **`right`** pointers of existing nodes in the tree.

Tree Traversals

Tree Traversals

- Often, we will want to "do something" with each node in a tree. Like linked lists, we can do so by **traversing the tree**. With the branching involved, this is a slightly more involved process than traversing a linked list!

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    - Pre-order traversal
    - In-order traversal
    - Post-order traversal

Tree Traversals

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- There are three main ways to traverse a binary tree:
    - Pre-order traversal
    - In-order traversal
    - Post-order traversal

- Due to the recursive nature of trees, all of these algorithms are most easily defined **recursively**.

Pre-order Traversal

- The algorithm for a pre-order traversal is defined as follows:
  - "Do something" with the current node
  - Traverse the left subtree
  - Traverse the right subtree

- For example purposes, let's have our "do something" to be printing the contents of the current node, which will allow us to print the overall tree.

Let's code it!

`preorderPrintTree()`

Pre-order Traversal

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- Output: **`pineapple coconut banana durian strawberry taro`**

In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
    - Traverse the left subtree
    - "Do something" with the current node
    - Traverse the right subtree

Let's code it!

`inorderPrintTree()`

In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
  - Traverse the left subtree
  - "Do something" with the current node
  - Traverse the right subtree

- Output: **`banana coconut durian pineapple strawberry taro`**

- Observation: The output of this traversal gives as all the values in alphabetical order. Is this a coincidence?
  - No! We'll see why this week!

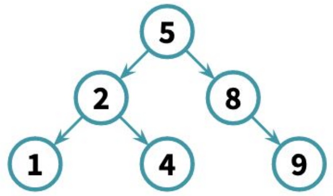
Post-order Traversal

- The algorithm for a post-order traversal is defined as follows:
  - Traverse the left subtree
  - Traverse the right subtree
  - "Do something" with the current node

Try it yourself!

`postorderPrintTree()`

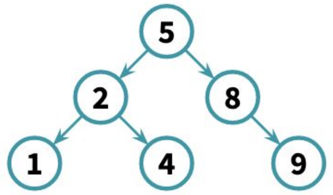
Pre-order



do something (aka cout)
traverse left subtree
traverse right subtree

5 2 1 4 8 9

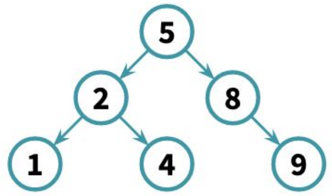
In-order



traverse left subtree
do something (aka cout)
traverse right subtree

1 2 4 5 8 9

Post-order



traverse left subtree
traverse right subtree
do something (aka cout)

1 4 2 9 8 5

Summary

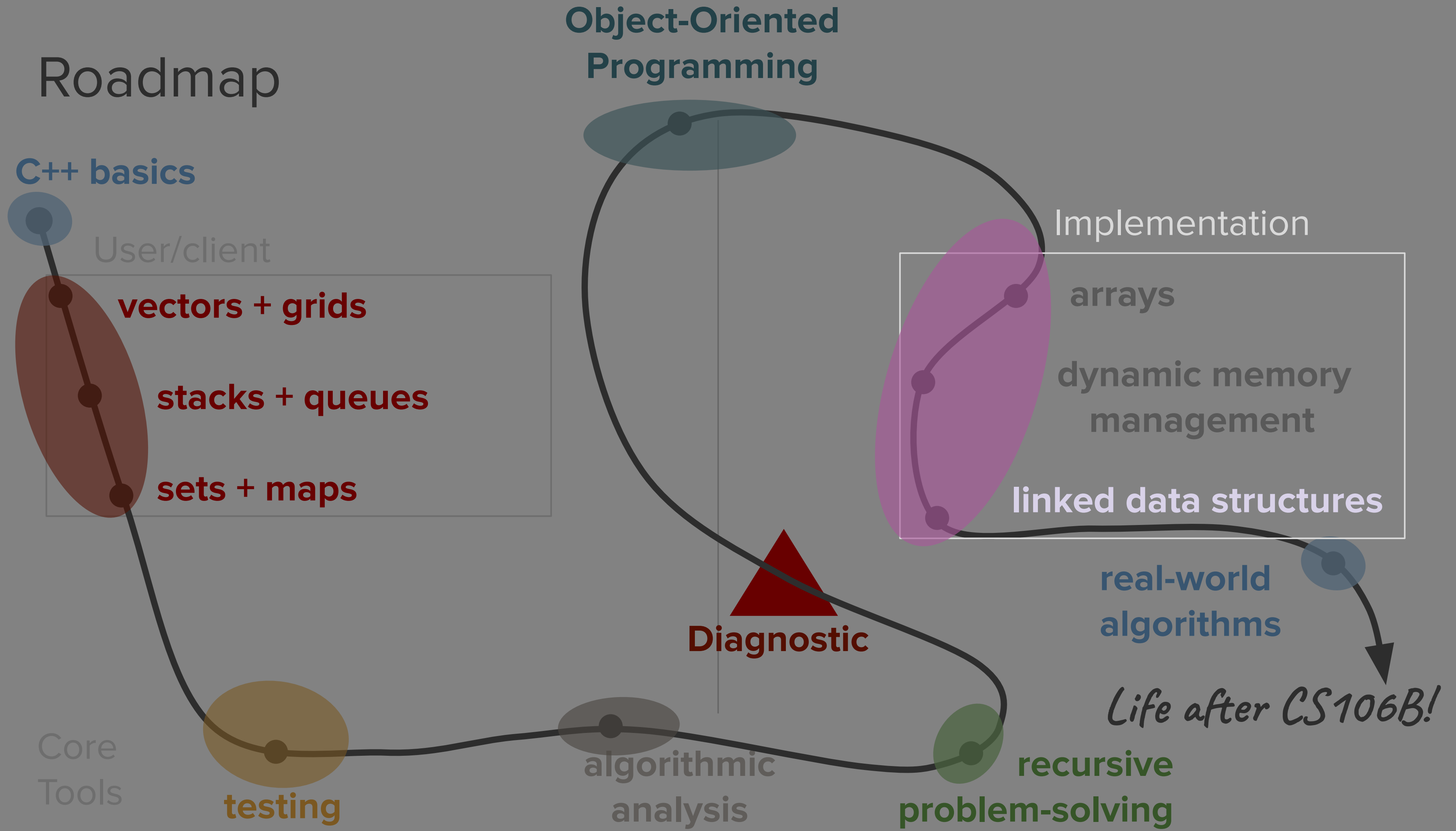
Trees Summary

- Trees allow us to organize information in a linked data structure such that the distance to any element is short, even if there are many elements.

- Trees organize nodes in a hierarchical manner, where each element contains connections to children nodes that exist "lower" in the tree.

- There are three main ways to traverse the nodes in a tree, and each type of traversal visits the nodes of the tree in a distinctly different order.

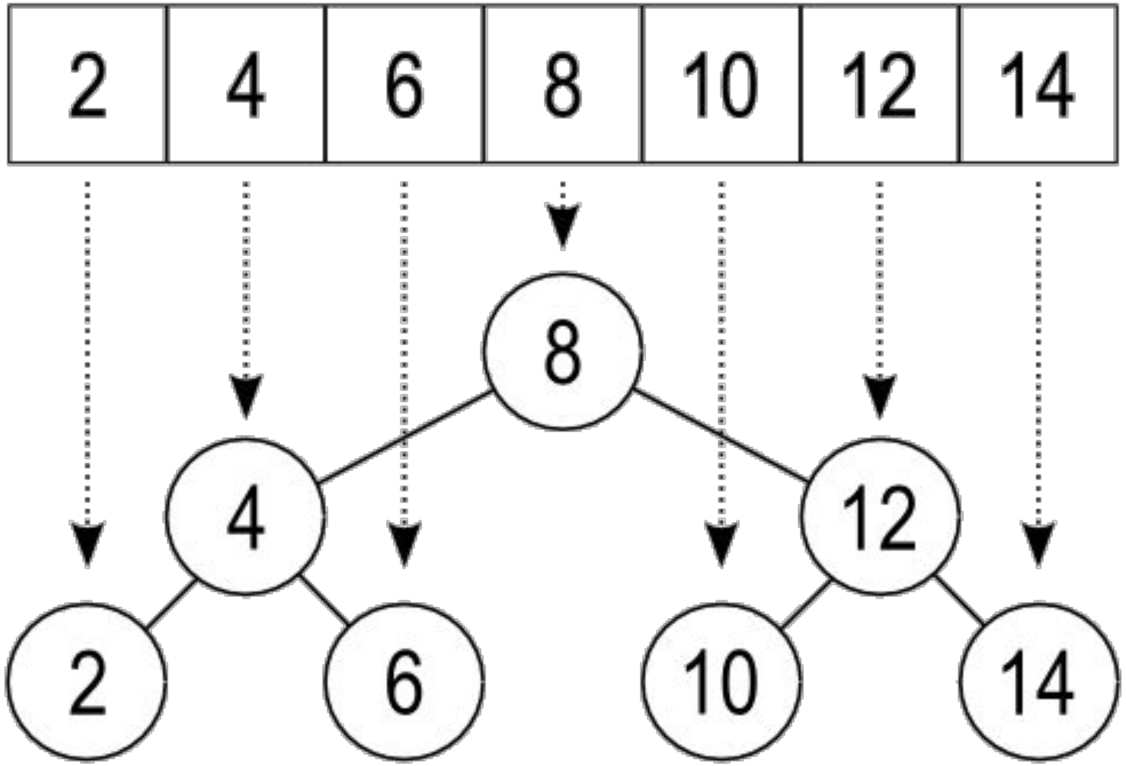
Roadmap


Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

What’s next?

Mannnnn, we spent a whole lecture on traversing a tree. When are we going to do something with a tree besides print it out??

Binary Search Trees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